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ABSTRACT.

A broadening of the RadioFrequency (RF) power spectrum coupled to tokamak plasma is necessary to occur in order to explain the existing experiments of Lower Hybrid Current drive (LHCD). The presented modeling shows that the Parametric Instability (PI) driven by ion sound quasi-modes produces in the scrape-off-plasma an important contribution to such spectral broadening. As effect of the quasi-linear interaction of the resulting LH spectrum penetrating in the bulk, the LH power fraction deposited in the plasma at the first pass results enhanced. Consequently, well defined LH deposition profiles are obtained when the ray propagation in toroidal geometry is taken into account. Considering the parameters of LHCD experiments of JET (Joint European Torus), and other machines as well, the PI growth rate is high enough for compensating the convective losses and broadening a fraction (≈10%) of the launched power spectrum. The LH spectral broadening is intrinsic to coupling RF power in LHCD experiments, and increases operating with higher plasma densities. As principal implication of such spectral broadening, experiments able evidencing the effects of a well-defined LH deposition profile, as those characterized by high electron temperature in the core and broad profile, are successfully interpreted. Useful experiments are the LHCD-sustained internal transport barriers of JET. The design of scenarios relevant to the modern fusion research program, which require the control of the plasma current profile in the outer half of plasma, can be properly achieved by considering the physics of the plasma edge for modeling the LH deposition profile and the q-profile evolution.

1. INTRODUCTION

The physical mechanisms that determine the LH deposition profile in realistic operating conditions should be considered for making the lower hybrid current drive (LHCD) [1-2] a robust tool for controlling the current profile in tokamaks. Since the LHCD effect [1-5] is based on the wave interaction with a tail of the electron distribution function of plasma, the assessment of the LH power $n_{\parallel}$ spectra that effectively propagate in the experiments is crucial for determining the deposition profile ($n_{\parallel}$ is the refractive index component in direction parallel to the confinement magnetic field). At this regard, a long-lasting debate is still open on the so-called spectral gap in LHCD, i.e., about the causes of broadening of the launched $n_{\parallel}$ spectrum, which is necessary to occur for explaining the available experimental data by means of the quasi-linear theory [3-6].

The approach of multi-radial pass produced by ray-tracing in toroidal geometry [6] was widely accepted as cause of spectral broadening, and utilized for modeling the LH deposition profile in experiments. A difficulty with such approach consists in the fact that the WKB approximation fails at the cut-off layers [3]. At these layers the LH waves are considered as optic waves, despite of their much longer wavelengths. The LH deposition profile might thus result arbitrary when undetermined reflections from the edge occur. Therefore, assuming that the multi-radial pass is the only cause of spectral broadening, the LH deposition profile might be not a well-defined feature of LHCD, which, thus, cannot be utilized as a robust tool for controlling the current profile. Conversely, there are
indications from experiments that well-defined LH-deposition profile are produced, as occurred, e.g., in the LHCD-sustained internal transport barriers (ITBs) of JET [7-10]. These barriers are characterized by high electron temperatures in the core with broad profiles, and most of the LH-driven current density is localized at two thirds of the minor radius. The observed ITB features are consistent with the hypothesis of an LHCD-sustained low/negative magnetic shear occurring in a layer close to the ITB radial foot [10]. Considering the multi-reflections as the only cause of the spectral broadening in LHCD, the precision of the LH-deposition profile results insufficient for finding consistency with the current profile supported by measurements, and with the observed features of the ITBs as well. Therefore, it seems that the multi-radial pass alone cannot bridge the spectral gap in LHCD. The role of the physics of the edge, which is supported by experimental observations in LHCD experiments of spectral broadening obtained by RF probes and by microwave reflectometry [10,11], should be considered for properly modeling the LH-deposition profile.

2. BROADENING OF THE ANTENNA SPECTRUM IN LHCD EXPERIMENTS BY THE PHYSICS OF THE EDGE

The physics of the edge appears evident when considering the whole scenario of the experiments that coupled LH waves to tokamak plasmas, including those aimed at heating the bulk ions [12]. Such experiments differ from the LHCD experiments essentially for the higher operating plasma densities, which is necessary for locating the cold lower hybrid resonant layer, \( \omega_{LH} \approx \omega_0 \), in the core, and meeting the mode-conversion of the launched Electron Plasma Wave (EPW) into an ion wave, which is collisionless absorbed by ion-cyclotron harmonic damping. The LHCD regime, which does not require mode-conversion, works instead at relatively low densities, \( \omega_{LH} \ll \omega_0 \), but higher than the EPW cut-off value: \( \omega_{pe} \geq \omega_0 \). The test of the ion heating scheme resulted impossible, since at the required high plasma densities the LH power does not penetrate in the bulk. The occurrence of Parametric Instability (PI) in the scrape-off plasma resulted the only effect on the plasma of the coupled RF power. An example of the PI spectra detected by RF probes is shown in Figure 1.

The phenomenology consists in a broadening around the operating line frequency and in several sidebands with a typical non-monotonic envelope [13-15]. Such spectra were interpreted in terms of a cascade of parametric instabilities [15]. PIs driven at very low frequencies by ion-sound quasi-modes deplete the pump and produce, in turns, a secondary LH pump with \( n_\parallel \) spectrum broader than that launched by the antenna. Such pump produces sidebands with maximum growth rates around the 10\(^{th}\) \( \Omega_{ci} \) harmonic, consistently with the observed typical non-monotonic envelop of sidebands. The full deposition of the coupled LH power at the edge is also explained in terms of strong quasi-linear absorption on the plasma particles due to the enormous LH spectral broadening, consistently with the observed ion tails from the plasma edge [5]. At the lower operating plasma densities of the LHCD experiments, the level of the ion-cyclotron frequency-shifted sidebands resulted generally much lower, with only a few sidebands monotonically decreasing at higher frequency shifts from the pump. The pump broadening is reduced too, but remains order of
magnitudes bigger than the frequency line width of the RF power sources utilized in the experiments. Therefore, the (non linear) physics of the plasma edge, which was not considered in the LH heating and CD schemes, produces a spectral broadening that depends on the operating plasma density. At relatively low densities the broadening is less pronounced but still persists, and, however, the RF penetration in the bulk is permitted. The spectral broadening must be however considered in every LH experiment, also considering that the LHCD experiments need that a spectral broadening occurs for working.

No behavior similar than the pump broadening was observed in the ICRH and ECRH experiments, which utilize electromagnetic waves. The LH waves are electrostatic, and thus the coherent motion of the particle in the wave field mainly carries the energetic flux. Therefore, there is not any a priori reason for neglecting the mode coupling of the LH waves with the thermal background particle motion at low frequencies ($\omega \ll \omega_0$). The parametric instability of a lower hybrid pump wave $\Phi_0 [-i(\omega_0 t - k_0 \cdot r)]$ is driven by a low frequency mode $\Phi [-i(\omega t - k \cdot r)]$ and grows by two sideband waves $\Phi_1,2 [-i(\omega_1,2 t - k_1,2 \cdot r)]$, where $k_2,1 = k \pm k_0$, $\omega_2,1 = \omega \pm \omega_0$ (selection rules). We assume $k_0 = \hat{k}_0 x + k_0 z$, $k_1,2 = \hat{k}_1,2 x + \hat{k}_1,2 y + k_1,2 z$, and utilize the relation $n = c \omega_0$ between refractive indexes and wavevectors. The plasma is modeled as a slab including the region of the edge close to the antenna mouth. The $x$ direction coincides with the (radial) direction of the plasma gradients, and $y$, $z$ correspond to the poloidal and the toroidal directions, respectively. The Vlasov-Poisson system is solved in slab plasma for LH coupled modes up to the 2nd order (referring to the perturbation of the low frequency mode). The relevant parametric dispersion relation for LHCD experiments is [16-18]:

$$\epsilon(\omega, k) - \frac{\mu_1(\omega_1, k_1, k_0, E_0)}{\epsilon(\omega_1, k_1)} - \frac{\mu_2(\omega_2, k_2, k_0, E_0)}{\epsilon(\omega_2, k_2)} = 0$$  \hspace{1cm} (1)$$

The solutions of Eq.1, $w$ is the complex frequency: $\omega = \omega_{Re} + i \gamma$, where $\gamma$ is the growth rate, and $\omega_{Re,1} = \omega_{Re} \pm \omega_0$. In Eq.1, $\epsilon$ is the dielectric function, $m_{1,2}$ are the coupling coefficients referring to the lower and the upper sidebands respectively, and are calculated considering the ion magnetized. The expression of the coupling coefficients is [18]:

$$\mu_{1,2} = \frac{\chi_e(\omega) - \epsilon(\omega)}{\chi_e(\omega)} \frac{\omega_{pi}^2}{\omega_0^2} \frac{\omega_{pi}^2}{4k^2 c_s^2} (1 + \frac{\omega}{k_c v_{the}} Z)^2 \sin^2 \delta_{1,2} \frac{u^2}{c_s^2}$$  \hspace{1cm} (2)$$

The coupling coefficients are derived in the limit for $\omega \leq k_{||} v_{the}$, which is satisfied by all the solutions of Eq.1 obtained considering typical parameters of the plasma edge of LHCD experiments performed in tokamak plasmas. In Eq.2, $\chi_e$ is the electron suscectivity, $c_s$ is the sound speed, $Z$ is the plasma
function. \( \delta_{1,2} \) are the angles which the perpendicular wavevectors of the lower and upper sidebands make with the perpendicular wavevector of the pump, i.e., \( \delta_{1,2} = \angle (k_{1,2\perp}, k_{0\perp}) \), and \( u = e k_0 \Phi_0 / m_e \omega_{ce} \). The angle \( \delta_{1,2} \) is an important parameter for determining the strongest PI channels, since it affects the convective loss. The solution of Eq. 1 is the complex frequency, \( \omega + i \gamma \) for a given wavevector component \( k_{\perp} \) of the low frequency mode that drives the instability, which is assumed as free variable. The plasma parameters of realistic LHCD experiments, supported by the measurements of plasma edge (spectroscopy of the edge and Langmuir probes) have been considered. For any run of the numerical calculation, one of the following parameters has been kept fixed: \( k_{||}, k_{0||}, \delta_{1,2}, \Phi_0, B_0, n_e, T_e \). In this way all the channels of PI are characterized by frequency and growth rates obtained by the analysis in the homogeneous plasma. The PI channel with highest growth rate results that driven by ion-sound quasi-modes. The effect of the gradient of the electron temperature reduces the growth rate for more internal radial position values.

The conditions for developing a parametric instability are produced by the convective loss due to the finite extent of the pump wave region and to the plasma inhomogeneity. The amplification factor should be \( \Lambda > 1 \) at the threshold condition, and \( \Lambda \approx 10 \) for producing a significant depletion of the coupled RF power into LH sideband waves that arise from the thermal noise. The details of this analysis are contained in Ref [10,11]. The depletion of the pump power has been calculated considering the classical reference of L. Chen and L. Berger [19]. The important result is that, for a typical LHCD experiment of JET [7, 9, 10], a fraction of about 10% of the pump power is deposited in the scrape-off layer on sidebands with \( n_{||} \approx 2.1 \div 2.3 \), as shown in Fig.2. This fraction decreases at increasing \( n_{||} \) (0.1% for \( n_{||} = 3 \)), due to increase of convective losses. For \( n_{||} > 3.5 \), no depletion occurs as the pump power density goes below the threshold. Thus, a cut-off in \( n_{||} \) of the LH power spectrum penetrating in the plasma bulk is determined. The LH-deposition profile has been calculated considering or not the contribution of the PIs to the spectral broadening, by utilizing a ray-tracing+2D-Fokker-Planck code [20], which retains the \( n/\) upshift due to ray propagation in toroidal geometry. The experiments of JET considered in the present analysis were performed with a small fraction of current drive, \( I_{LHCD}/I_p \approx 30\% \), utilizing LHCD both during the current ramp-up, and at the end of the ramp-up (\( I_p = 2.3 \text{MA}, B_T = 3.3 \text{T} \)), in combination with the main heating power (16MW of neutral beam and 5MW of ion-cyclotron resonant heating), [7,9,10]. The significant effects of the LHCD on the ITBs, resulting radially broad and long-lasting (about 40 \( t_E \)), can be reasonably explained only by considering the effect of the physics of the edge. Indeed, the LH-deposition profiles obtained considering or not the physics of the edge have been inputted, with experimental kinetic profiles and magnetic measurement data, in the JETTO code [21] for modeling the evolution of the q-profile. Considering the phase of the current ramp-up, the LH deposition profiles and the respective q-profiles are shown in Fig.3. The effects in the LH-depositions when retaining the minor changes due to measurement uncertainty of the inputted kinetic profiles are also shown in the figure. It is evident that only including the effect of the physics of the edge a more precise deposition profile is obtained, with most of the current driven at
about half radius. In a further simulation performed considering LHCD during the main heating phase, in which broader Te profiles occur, the peak of LH deposition moves to about two thirds of the minor radius. The corresponding q-profiles are shown in the Fig.4 (case of the experiment with $P_{\text{LHCD}} = 2.2\text{MW}$). In the experiment, the radial foot of the LHCD-sustained ITBs is produced close to the radial position of the layer with low/negative shear. The low shear condition is lost at $t \approx 10\text{ s}$, consistently with the ITB collapse. Steady state ITBs are expected to occur, instead, operating with double LHCD power. These results support the hypothesis that a low shear layer stabilizes the turbulence, thus improving the local confinement and producing the observed ITB behavior [22-26].

The PI produces a similar spectral broadening in LHCD experiments performed in different machines [10, 11]. The typical parameters relevant to the PI-driven quasi-mode are: $k \perp \approx 10-15\text{cm}^{-1}$, $\omega/2\pi \approx 0.2$ – 1MHz, consistently with the frequency broadening measured in the LH range by RF probes, and in the density fluctuation range by microwave reflectometry [27,11]. Such common behavior is determined by the circumstance that all the LHCD experiments meet similar operating conditions, in the operating frequencies and scrape-off parameters, which determine mainly the growth rates and the launcher coupling performance as well. These conditions are: $n_0||$ in the range 1.5–3, and layers with $\omega_{\text{pe}}/\omega_0=1$ are located near the effective antenna-plasma interface (as necessary for launching the slow electron plasma wave); $\omega_{\text{pe}}/\omega_0$ of the order of ten, or more, in the layers close to the last closed magnetic surface, as typically obtained in JET and in other tokamaks as well [28,29]; the electron temperatures are in the range from a few eV to 100 eV; the scrape-off radial dimensions lye typically in the range of 3-6cm.

Figure 5 shows the LH deposition profile modeled considering the physics of the edge for an experiment utilizing LHCD on FTU [28,29]. The peak of the deposition profile is close to the maximum of the emission of the FEB camera (fast electron Bremsstrahlung). However, the relatively narrow electron temperature profile does not allow performing a satisfactory test of the precision of the LH deposition profile. The electron temperature profile of the FTU experiment is indeed much narrower than that of the aforementioned experiment of JET. The layer with 2keV is located at about one thirds of the minor radius. Such circumstance produces a deposition reasonably located inside the inner half of plasma utilizing whatever LHCD model.

CONCLUSIONS

The conditions necessary for the occurrence of the spectral broadening induced by parametric instabilities coincide with those necessary for coupling LH power to tokamak plasmas. The parametric dispersion relation available by classical works of the literature and the convective losses due to finite extent of the pump and plasma inhomogeneity have been considered for carrying the numerical computation of frequency, growth rates, amplification factor and pump power depletion of the PIs. The ion-sound-driven PI represents the most important channel for LHCD experiments. The present model explains the existing results on long-lasting ITBs sustained by LHCD. A layer with low magnetic shear is produced in the outer half of plasma, consistently with both the radial foot location
and with the time duration of ITB. Conversely, by considering only the multi-radial reflections to bridge the n// gap, the experimental results are not interpreted with sufficient accuracy.

Due to the dependence of the PI growth rate on the electron temperature, a strong reduction of the PI growth is expected operating with low recycling/higher electron temperature at the edge (e.g., by vessel Lithium-coated [30,31]). In these conditions, LHCD operations at operating plasma densities higher than in the present experiments would be possible. In addition, a proper tailor of the launched n// antenna spectrum (electronically achievable) would possibly allow a successful control of the LH deposition and of the magnetic shear profiles in the plasma.

The use of the proposed LHCD model as a predictive tool will allow the design of experimental scenarios requiring the control of the q-profile for improved stability and confinement in steady-state and advanced-tokamak regimes.

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Figure 1. RF probe spectra in the LH experiment aimed at ion heating in FT.

Figure 2: Parametric instability-induced spectral broadening.

Figure 3: LH current density profiles and respective modeled q-profiles compared with that supported by MSE measurements.
Figure 4: \(q\)-profile evolution for ITB experiments of JET. Interpretive modeling with 2.2MW of LHCD power. Predictive modeling with 5.5MW of LHCD power. \(\rho\) is the normalized flux co-ordinate.

Figure 5: Modeling of the LH deposition profile compared with the FEB camera profile for an electron ITB experiment produced utilizing 1.5MW of LHCD power on FTU. \(\rho\) is the flux co-ordinate.