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Evaluation of Spectral Unfolding Techniques for Neutron Spectroscopy

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ABSTRACT.
The precision of the JET installations of MAXED, GRAVEL and the L-curve version of MAXED has been evaluated by using synthetic neutron spectra. We have determined the number of counts needed for the detector systems NE213 and MPR to get an error below 10% of the MAXED unfolded neutron spectra is determined to be ~10^6 and ~10^4, respectively. For GRAVEL the same number is ~10^7 and ~3·10^4 for NE213 and MPR, respectively.

1. INTRODUCTION
All neutron spectrometers measure physical quantities, e.g. light yield, time of flight, etc., which in turn are processed and interpreted as a neutron spectrum. Unfolding can be used for processing the measured data and obtaining the neutron spectrum. The advantage of the unfolding method is that the analysis can be totally unbiased. Several unfolding codes have been developed, e.g. MAXED [1], GRAVEL [2], etc. This paper evaluates how well these codes can unfold data with detector response functions (DRF) of two neutron diagnostic techniques, namely the Magnetic Proton Recoil (MPR) and of a liquid scintillator, NE213.

2. METHOD
We have setup a method to evaluate the performance of different unfolding techniques using synthetic neutron spectra of various shapes. In this paper, we limit our study to a Gaussian shaped neutron emission, corresponding to purely thermal plasma with an ion temperature of 20 keV. The DRFs in the study are those used at JET for analysis purposes. The target $\chi^2$ used for MAXED is 1.1.

The synthetic neutron spectra are folded with a spectrometer response function resulting in an ideal instrument response without statistical fluctuations. Poisson errors are added to this to simulate a realistic measurement. The different unfolding codes are applied to these Poisson perturbed data, resulting in an unfolded spectrum. A quality value, $q$, is calculated for each unfolded spectrum

$$q = \frac{\sum |unf_i - synth_i|}{\sum synth_i}$$

where $unf_i$ is the unfolded spectrum’s $i$:th bin value and $synth_i$ is the same for the original synthetic spectrum. The quantity $q$ shows how well a specific unfolding technique can recover the original neutron spectrum. This process is repeated many times and for each unfolded spectrum values of $T$ and $q$ are extracted. In this way we can assemble a frequency distribution of $q$ and $T$, which are studied to evaluate the performance of the unfolding technique.

3. STARTING GUESSES OF UNFOLDING CODES
For a proper unbiased evaluation the unfolding should start with a uniform initial guess. If the starting guess is altered from uniform to peaked the result of the unfolding is changed, introducing
a systematic error. This is illustrated in Fig.1, where the unbiased analysis gives a distribution of $T$ around the original synthetic data, while a peaked initial guess gives a distribution of $T$ skewed towards lower values than the original. Thus, the results of the unfolding are dependent on the starting guess as shown in Fig.1. This effect decreases with the number of counts.

4. MAXED AND GRAVEL

Figure 2 illustrates the performance of three different unfolding techniques for different number of counts. For MAXED, the synthetic data is reproduced with a $q$ of about 10% with $\sim 10^4$ and $\sim 10^6$ counts for the MPR and NE213, respectively. For GRAVEL, the synthetic data is reproduced with a $q$ of about 10% with $\sim 3 \cdot 10^4$ and $\sim 10^7$ counts for the MPR and NE213, respectively. A typical Gaussian DRF, like MPR, is easier to analyse for the unfolding codes.

5. MAXED L-CURVE

An inherent problem of unfolding techniques is the risk of over fitting, i.e. using too low minimisation quantity, for example $\chi^2$ in the case of MAXED. One way of dealing with such problems is to use the so-called L-curve method [3]. The L-curve implementation of MAXED (MAXED-L) is designed to find the best $\chi^2$ for the problem and thus avoiding the risk of over fitting. To study this we have used ordinary MAXED to investigate if MAXED-L is finding the right $\chi^2$.

By using the method described above and using different target $\chi^2$ the corresponding $q$ curves can be obtained as a function of the number of counts in the synthetic data (Fig.3 left frame). The curve with the lowest $q$ given a number of counts correspond to the best $\chi^2$ to use. The best $\chi^2$ to use for a given number of counts is plotted in Fig.3 (right frame, solid line) together with the $\chi^2$ calculated by MAXED-L. The unevenness of the curve is an effect of insufficient number of MAXED runs per data point in the analysis presented in the left frame of Fig.3. MAXED-L seems to fail to find the best $\chi^2$ of the problem, except for low number of counts.

CONCLUSIONS AND OUTLOOK

The result of MAXED is dependent of the initial guess of the unfolding. This implies that it is important to show what starting guess is used when unfolding is done.

The NE213 detector system needs about 100 times more counts than the MPR system to achieve the same level of error.

The JET installation of MAXED-L seems to underestimate the best target $\chi^2$ to use.

There are alternatives to MAXED and GRAVEL for finding the neutron spectra, e.g. MFR [4] and neural networks. MFR is still under development and therefore not included in this study. A future study would benefit of including, at least, these two unfolding techniques and using more DRFs of other instruments, e.g. diamonds or a hypothetical 14MeV Time of Flight spectrometer.
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REFERENCES

Figure 1. Left frame: Neutron spectra from an unfolding using MAXED on synthetic MPR data with 1000 number of counts. Green curve is the original synthetic spectrum, i.e. the correct answer. Solid blue and red curves are the averages of 100 runs (with standard deviations given by the dashed lines of the same color). Solid red curve is the result of unfolding with a uniform initial guess and solid blue curve corresponds to a Gaussian starting guess (of 1keV). Right frame: The relative temperature difference distribution of the unfolded and the synthetic spectrum using a uniform initial guess (red curve) and the same for blue curve, but for a Gaussian initial guess.
Figure 2: The quality value, $q$, as a function of number of counts for three different unfolding techniques, namely MAXED (black), GRAVEL (green) and MAXED-L (red). Left panel for MPR and right panel for NE213.

Figure 3: Left frame: Curves of the average $q$ values for 100 MAXED unfolded spectra for a number of target $\chi^2$ values as a function of the number of counts in the synthetic data. Right frame: The best $\chi^2$ according to left frame as a function of number of counts (line) and the $\chi^2$ suggested by MAXED-L (stars).