Nonlinear and Kinetic Effects on Resistive Wall Mode Stability
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1. INTRODUCTION
The ultimate performance limit in advanced tokamak scenario operation is often set by the Resistive Wall Mode (RWM). However, operation above the stability limit for RWM onset is important in order to achieve economically-attractive plasma performance in advanced tokamak regimes, which aim at steady-state operation with high plasma pressure and a large fraction of non-inductively driven current. Various experiments have shown that the RWM can be stabilised in such a way that the plasma can operate above the no-wall $\beta$-limit [1, 2]. Recent experiments with nearly balanced neutral beam injection [3, 4] have found a critical velocity for RWM stabilisation well below that found in previous magnetic braking experiments [5]. There is naturally much interest in understanding the passive stabilisation occurring from kinetic effects which is thought to underlie these results. Various models have been presented to explain the RWM damping due to kinetic effects, such as sound-wave damping [6], ion Landau damping [7], or the precessional drift resonance with thermal ions [8].

2. MODELLING KINETIC EFFECTS
The kinetic effects influencing the stability of the Resistive Wall Mode (RWM) have been investigated by applying the HAGIS drift kinetic code [9] to calculate the change in the potential energy of the mode in the presence of thermal and energetic particles.

Analytic theory developed for large aspect ratio circular plasmas shows that the contribution from the perturbed distribution function, found by solving the drift kinetic equation, leads to a kinetic potential energy of the form

$$\delta W_K \sim \sum_{l=-\infty}^{\infty} \frac{\omega - n\omega - l\omega_s}{\omega - n\Omega - l\langle \phi_E \rangle_{md} - I(\theta)}$$

where $\omega$ is the mode frequency, $\Omega$ is the plasma rotation frequency, $\phi_E$ has been redefined to be in a frame rotating with the $E \times B$ velocity, the dots represent the time derivative and $\langle ... \rangle$ is the orbit average, $\langle \phi_E \rangle_{md} = \langle \phi_E \rangle - \Omega_{ExB}(r)$ and $\omega_s$ is the diamagnetic frequency, $\omega_{si,e} = (m_{i,e}/Ze)\frac{\partial f_{i,e}}{\partial E}|_{\psi} \frac{\partial E}{\partial \psi} + \Omega_{ExB}$ where $E = v^2/2$.

The areas of strongest interaction indicate that the particles in that region of phase space have appropriate toroidal and poloidal frequencies to maximise equation 1. By considering the interaction arising from the precessional drift resonance of the trapped thermal ions, the kinetic damping can be assessed as the plasma rotation frequency varies. Figure 1 shows $\delta W_K$ as a function of $\Omega$ for JET Pulse No: 68875. The thermal particle distribution function is considered to be isotropic with respect to pitch angle and Maxwellian with respect to energy. Evidently, the plasma rotation plays a significant rôle in determining the kinetic stabilising effect upon the mode. The wave-particle interaction is strongest when the Doppler shifted mode frequency is approximately the same as the average precession drift frequency of the ensemble of trapped particles. This may partially explain how recent experiments have found the RWM stable at very low rotation when $\beta > \beta_{\infty}$ [3, 4].

The growth rate of the RWM can be formulated in terms of MHD perturbed energy as [8]: where
\[ \delta W \text{represents the sum of the plasma and vacuum energy with and without a wall respectively and} \]

\[ \delta W_K \text{ is the kinetic contribution to the plasma energy. HAGIS is a particle-orbit code following the} \]

\[ \text{guiding centre motion of the particles. Consequently it includes any finite orbit width effects which} \]

\[ \text{have been neglected in previous studies [8].} \]

\[ \gamma_{\tau W} = - \frac{\delta W_0 + \delta W_K}{\delta W_0 + \delta W_K} \]

\[ (2) \]

The analysis is carried out for typical JET high-\( \beta \) plasmas. For Pulse No: 68875 at \( t = 5.0s \), the no-wall b-limit is found to be \( \beta_{N0} = 2.7 \). The static eigenfunction is computed using the MISHKA-1 code [10] for the no-wall case, and then input into HAGIS together with the equilibrium generated by the HELENA code [11]. The growth rate of the RWM, \( \gamma_{\tau W} \), calculated using equation 2 is illustrated in figure 2. Including the kinetic effects increases the RWM stability limit by over 10\%, beyond the no-wall limit. The strongest kinetic damping of the RWM arises due to mode resonance with the precession motion of the trapped thermal particles [12, 13].

3. NONLINEAR MODELLING

The JOREK code [14, 15] is being developed in order to explicate the nonlinear interplay between the RWM, applied magnetic fields and other plasma parameters, such as plasma rotation. The nonlinear behaviour of the RWM has been studied in general toroidal geometry for typical JET current and pressure profiles by solving the reduced MHD equations. The hot core plasma, with density \( r_0 \) is surrounded by a cold plasma with low density, \( r_w \), and high resistivity. Outside the cold plasma is a resistive wall at \( r = r_{rw} > a \) with resistivity \( h_w \), which in turn is surrounded by a vacuum and an ideal wall at \( r = r_w > r_{rw} \). Figure 3 shows that whilst the linear growth rate of the RWM for \( r_{rw} = 1.3a \) depends upon the density of the cold temperature, high resistivity plasma surrounding the core plasma, the nonlinear saturation level does not, in good agreement with cylindrical modelling [16, 17]. If \( \rho_w/\rho_0 \) is dropped below \( 10^{-3} \), the linear growth rate does not change, and is equal to the inverse wall time. JOREK simulations also show that when the resistive wall is close to the plasma, \( r_{rw} < 1.2a \), neither the linear growth rate nor the energy saturation level of the RWM varies with \( \rho_0/\rho_w \).

Nonlinear simulations of RWms have also been performed for JET profiles at artificially inflated plasma pressure such that the equilibrium is found to be unstable to both the \( m/n = 2/1 \) RWM and an internal 3/2 mode. Figure 4 illustrates that in the presence of the RWM, the 3/2 mode is significantly destabilised in the nonlinear phase, due to the change in current profile at the \( q = 3/2 \) surface resulting from the growth of the \( n = 1 \) mode. The dominant poloidal harmonic is found by examining the perturbation in the poloidal plane. This example illustrates the nonlinear mode coupling which can be studied with JOREK, even using reduced MHD which neglects some important tearing mode physics. Such interaction between the RWM and other core MHD instabilities may help to shed light on recent observation of RWM coupling with fishbones [18] and ELMs [19].
SUMMARY AND FUTURE WORK

Recent improvements in numerical modelling have allowed investigation of RWM stability including kinetic damping and nonlinear evolution. Drift kinetic modelling with the HAGIS code has been used to assess the kinetic damping of the RWM in high-\(\beta_N\) JET plasmas including all wave-particle interactions rigorously. The kinetic damping increases the no-wall b limit, primarily due to the interaction between the RWM and the trapped thermal ions. Nonlinear MHD modelling with the JOREK code has shown that saturation level of the RWM is independent of the density of the high resistivity plasma surrounding the plasma core, and that the 2/1 RWM unstable for typical JET profiles destabilises an internal mode in the nonlinear phase.

We intend to develop the JOREK code further in order to consider how the RWM interacts with externally applied fields. In order to do this, the full MHD equations will be implemented, arbitrarily shaped walls will be included, sparse vacuum region grids will be adopted for \(r > r_{rw}\), the ideal wall removed and the necessary boundary conditions for external fields added.

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REFERENCES

Figure 1: $\Re(\delta W_K)$ as a function of the plasma toroidal rotation. The influence of the kinetic damping changes significantly with rotation.

Figure 2: The $\beta$-limit of the RWM is increased by more than 10% for JET Pulse No: 68875 when the kinetic damping terms are included.

Figure 3: Time evolution of the magnetic energy of the RWM for different $\rho_w$.

Figure 4: Time evolution of the magnetic energy of the $3/2$ and $2/1$ components. The $3/2$ mode is destabilised in the nonlinear phase.

[18]. M. Okabayashi et al. 22nd IAEA FEC, Geneva *EX/5-2* (2008)