Model for Screening of Resonant Magnetic Perturbations by Plasma in a Realistic Tokamak Geometry and its Impact on Divertor Strike Points
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\textsuperscript{*}See annex of F. Romanelli et al, “Overview of JET Results”, (Proc. 22\textsuperscript{nd} IAEA Fusion Energy Conference, Geneva, Switzerland (2008)).
ABSTRACT
This work addresses the question of the relation between strike-point splitting and magnetic stochasticity at the edge of a poloidally diverted tokamak in presence of externally imposed magnetic perturbations. More specifically, ad-hoc helical current sheets are introduced in order to mimic a hypothetical screening of the external resonant magnetic perturbations by the plasma. These current sheets, which suppress magnetic islands, are found to reduce at the same time the amount of splitting expected at the target, which suggests that screening effects should be observable experimentally.

1. INTRODUCTION
An axisymmetric, single null, poloidally diverted tokamak has two strike-lines where the plasma hits the divertor targets: one on the High Field Side (HFS) and one on the Low Field Side (LFS). In presence of a non-axisymmetric magnetic perturbation, these strike-lines are replaced by spiralling patterns. If an experimental profile (e.g. D$_a$ or InfraRed [IR]) is taken along the radial direction at a given toroidal location, the strike-points are then observed to split. Such a splitting is for instance commonly observed during locked modes [3]. In presence of external Resonant Magnetic Perturbations (RMPs) a splitting may also be expected. On DIII-D, the splitting is observed during Edge Localised Mode (ELM)-suppressed discharges using n = 3 perturbations from the I-coils [1,2]. It is important to notice that the splitting is seen much more clearly on particle flux (D$_a$) profiles than on heat flux (IR) profiles, at least in low collisionality experiments [1] (at high collisionality, the splitting is however clearly observed on the heat flux [2]). Recently, DIII-D also reported on splitting observations (both on heat and particle fluxes) in L-mode plasmas [4]. JET (using the Error Field Correction Coils (EFCCs)) and MAST (using n = 3 perturbations from the ELM control coils [6]) find consistent effects on the heat flux profiles: the splitting is observed in L-mode but not in H-mode [5].

In the field of ELM control by RMPs from perturbation coils, one major question is to know whether the RMPs stochastize the magnetic field at the edge of the plasma. Vacuum modelling suggests that it is the case. Furthermore, studies based on the vacuum field assumption [8] have led to a design criterion for the considered ITER ELM control coils [9]. However, two important elements cast doubt on the stochastization of the magnetic field. The first one is the absence of a degradation of the electron temperature gradient in the edge transport barrier, which would be expected in presence of a stochastic field. The second one is the strong rotational screening effect [11,12] found in simulations of the DIII-D ELM suppression experiments [13,14].

In this paper, we analyse the possible consequences of the rotational screening on the strike-point splitting in order to assess whether screening effects may explain the absence of a clear splitting of the heat flux profiles in some experiments, in particular in the DIII-D, JET and MAST H-mode discharges referred to above.

2. Modelling and theoretical understanding of the strike-point splitting
Under a non-axisymmetric perturbation the magnetic separatrix splits into two surfaces: the stable
and unstable manifolds of the X-point. The stable (resp. unstable) manifold is the set of field lines that approach asymptotically the X-point when followed in the direction of (resp. opposite to) the magnetic field. The manifolds are of interest to experiments because they delimit the volume of field lines which reach the plasma core. Their intersections with the divertor plates thus define areas (divertor footprints) where high heat and particle fluxes are carried from the plasma core along the field lines [16]. Those areas take typically the form of spirals of high temperature and particle recycling around the original (unperturbed) divertor strike point.

The divertor footprints can be visualized by plotting a map of the connection length on the divertor plates (a laminar plot) [7]. The connection length is the distance (measured as the number of toroidal turns) needed to reach the wall again by following a field line starting at a given position. Field lines with large connection lengths remain in the plasma for many turns and carry high fluxes from the hot plasma core. The extent of the footprint can be approximated analytically using the Melnikov function [17] whose maximum is the difference of $\psi$ between the unperturbed strike point and the tip of the footprint [18]. When the perturbation has one dominant toroidal mode further simplification is possible and the difference of $\psi$ can be expressed using a single number – the one-mode Melnikov integral $\tilde{M}_n$ [18].

An example of the laminar plot and the stable manifold is shown on Fig.1 (left plot) for an equilibrium predicted for the COMPASS tokamak in the case of a magnetic field of 1.2T, single-null, low triangularity (SND) geometry and heating by one co-injected neutral beam [19]. The $n = 2$ perturbation is imposed by the existing perturbation coils whose description can be found in [20].

3. PHYSICS-MOTIVATED METHOD FOR TAKING SCREENING CURRENTS INTO ACCOUNT

3.1. COORDINATE SYSTEM AND RESONANT FIELD COMPONENTS

We use an $(s, q, \theta^*)$ system of equilibrium coordinates, where $s = \left| \frac{(\psi - \psi_{axis})}{(\psi_{sep} - \psi_{axis})} \right|^{1/2}$ (with $\psi$ the poloidal magnetic flux), $q$ the geometric toroidal angle and $\theta^*$ the corresponding straight field line poloidal angle $\left| \frac{dq}{d\theta^*} \right| = \text{const} = q$ (i.e. such that along a field line, with the safety factor).

Magnetic islands are known to arise from the component of the magnetic perturbation which is perpendicular to the equilibrium flux surfaces. We characterise the latter by the quantity $b^1 = \vec{B} \cdot \vec{\nabla} s / \vec{B} \cdot \vec{\nabla} \psi$. It can be shown that its Fourier components $b_{mn}^1$ are directly related to the half-width of the magnetic islands and moreover they are proportional to a function which is a generalization of the one-mode Melnikov integral $\tilde{M}_n$ (the Melnikov-like function) [18].

3.2. MODEL OF THE SCREENING CURRENTS

Screening currents are modelled under the following assumptions:

1) They are radially localised on infinitesimally thin layers around the resonant surfaces:

$$j = \sum_{(m,n) \in S} \delta(s - s_{mn}) j_{mn},$$

where $\delta$ is a Dirac delta and $S$ is the set of screening surfaces, defined by $q(s_{mn}) = m/n$. 

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2) They are parallel to the equilibrium field lines: \( \vec{J}_{mn} = \frac{J_{mn}}{B_{eq}} \cdot \vec{B}_{eq} \).

3) They are divergence-free: \( \nabla \cdot \vec{j} = 0 \), which implies that \( \alpha_{mn} = \frac{J_{mn}}{B_{eq}} \) is constant on a field line. The first assumption corresponds to the fact that the current density is generally localised in a thin resistive (or visco-resistive) layer around the resonant surface [11,12]. The second assumption is a consequence of the flute ordered (\( k_i \ll k_\perp \)) nature of the physics at the resonant surface [11][12]. The third assumption expresses quasi-neutrality.

The real quantity \( \alpha_{mn} \) can be expressed as a Fourier sum:

\[
\alpha_{mn} (\theta^*, \varphi) = \left( \sum_{m' \geq 0, n' \geq 0} \alpha_{mn}^{m'n'} \exp[i(m'\theta^* + n'\varphi)] \right).
\]

According to our third assumption and restricting ourselves to the case of a single \( n \) for the screening currents, the only non-zero component is . Thus, the screening current density can be expressed as

\[
j = \left( \sum_{(m,n) \in S} I_{mn} \cdot \vec{J}_{mn,0} \right),
\]

with \( I_{mn} = \beta_{mn} B_{ref}/j_{ref} \) and the base currents

\[
\vec{j}_{mn,0} (s, \theta^*, \varphi) \approx \frac{j_{ref}}{B_{ref}} \delta(s-s_{mn}) \exp[i(m\theta^* + n\varphi)] \cdot \vec{B}_{eq}
\]

is an arbitrary value expressing the choice of base current amplitudes relative to the magnetic field strength.

### 3.3 COUPLING MATRIX

We note \( \vec{B}_{mn,0} \) the field created by the base resonant current \( \vec{J}_{mn,0} \). It is a complex quantity, just as \( \vec{J}_{mn,0} \) and we calculate it numerically. The corresponding \( b^1 \) component, denoted \( b_{0mn}^1 \), is then Fourier transformed at each resonant surface \( q = m'/n' \) in order to obtain the resonant components \( b_{mn',0}^{1mn} \). Thus, \( b_{mn',0}^{1mn} \) designates the resonant part, on the \( q = m'/n' \) surface, of the \( b^1 \) created by a resonant current \( J_{mn,0} \) located at the \( q = m/n \) surface. Thanks to the toroidal symmetry of the field equations \( b_{m'n',0}^{1mn} \) in we have , otherwise \( m/n' b_{m'n',0}^{1mn} = 0 \).

The plasma response field corresponding to the total current \( \vec{j} \) as given by Eq. (1) is \( \vec{B}_{plasma} = \Re(\sum_{(m,n) \in S} I_{mn} \vec{B}_{mn}^0) \) whose resonant \( b^1 \) components are \( b_{m'n',0}^{1mn,\text{plasma}} = \sum_{m} b_{m'n',0}^{1mn} \cdot I_{mn}/2 \) and \( b_{m'-n,0}^{1mn,\text{plasma}} = \sum b_{m'n',0}^{1mn} \cdot I_{mn}/2 \). The RHS is the product of the matrix \( b_{mn,0}^{1mn} \) with subscripts \( m \) and \( m' \), which we call the coupling matrix, by the current vector \( I_{mn} \).

### 3.4 CALCULATION OF THE SCREENED FIELD BY INVERSION OF THE COUPLING MATRIX

To determine the \( I_{mn} \) coefficients one needs an assumption about the character of the plasma response, i.e. if it amplifies or screens the perturbation and by what amount. In the following we
assume an efficient screening which completely eliminates magnetic islands at the rational surfaces in the pedestal region, i.e. the resonant Fourier components $b^{1}_{mn}$ of the total magnetic field are zero.

The procedure to obtain the screened field begins with the calculation of the coupling matrix, for a given choice of the set of screening surfaces $S$. Independently, the vacuum RMP spectrum $b^{1}_{m'n', vac}$ is calculated from the coil geometry. The screening current distribution $I^{\text{screen}}_{mn}$ is obtained by solving $\sum_n b^{1}_{m'n,0} \cdot I^{\text{screen}}_{mn}/2$ and $b^{1}_{m'n,vac}$ i.e. by inverting the coupling matrix. The full, screened field is then obtained as $B^{\text{full}} = B^{0}_{mn} + \Re(\sum_{(m,n)\in S} I^{\text{screen}}_{mn} B^{0}_{mn})$. It is easy to verify that it satisfies the property $b^{\text{full}}_{mn}$ for any $(m,n) \in S$.

If the plasma is represented as a straight cylinder, it is poloidally symmetric. The geometric poloidal angle $\theta$ is equal to $\theta^*$ and the coupling matrix is diagonal: $b^{1}_{m'n,0} = 0$ for $m' \neq m$. Cylindrical models are often used to simulate interaction of magnetic perturbation with plasma [13,14,15] because of the numerical simplification arising from the poloidal symmetry. For a realistic geometry we may recalculate the screening currents $I^{\text{screen}}_{mn}$ using only the diagonal terms of the coupling matrix: $I^{\text{screen}}_{mn, \text{diag}} = -2b^{1}_{mn, vac}/b^{1}_{m'n,0}$. The ratio between $I^{\text{screen}}_{mn}$ calculated using the full matrix and $I^{\text{screen}}_{mn, \text{diag}}$ quantifies the inaccuracy of the cylindrical approximation. If for example the ratio is lower than 1, it means that in a cylindrical model larger induced currents will be needed to screen the perturbation than in the realistic geometry, thus in the cylindrical model the penetration of the field will appear easier than in reality.

3.4. NUMERICAL EXAMPLES

We chose two cases to illustrate the method and to show the effect of screening on footprints: the COMPASS case described above and an equilibrium from an ELM control experiment on JET with $n = 2$ perturbation of the EFCCs [5] (Pulse No: 79729 at 19.38s). For both cases we first calculate the screening field needed to cancel the perturbation on a single outer surface ($q = 7/2$ for COMPASS, $q = 8/2$ for JET). In those cases the coupling matrix is trivial, with one element. Then we calculate the screening field choosing four screening surfaces with $q = 4/2, 5/2, 6/2$ for COMPASS, $q = 5/2, 6/2, 7/2, 8/2$ for JET. The $n = 2$ mode of the screening field $b^{1}_{\text{screen}}$ as a function of $\theta$ on the outermost screening surface is shown on Figure 2 for the four cases (each equilibrium with one and four screening surfaces). The screening field of one screening surface is distributed all over the resonant surface, due to the helical structure of the screening current, while for four screening surfaces the screening field is mostly localized on the LFS. Both COMPASS and JET show this effect. It shall be noted that the vacuum field is also localized at the LFS because of the position of the coils in both tokamaks, so the screening field of four currents is more similar to the vacuum field than the field of a single current. The ratio $I^{\text{screen}}_{m = 8 n = 2}/I^{\text{screen}}_{m = 8 n = 2, \text{diag}}$ for JET is 0.50, for COMPASS $I^{\text{screen}}_{m = 7 n = 2}/I^{\text{screen}}_{m = 7 n = 2, \text{diag}}$ is 0.48, and similar values for other surfaces.

4. IMPACT OF SCREENING CURRENTS ON THE SPLITTING

For the $b^{1}_{0} \text{ field created by the resonant current } j_{mn,0}$ we may compute the one-mode Melnikov
integral at the separatrix which we will note $\tilde{M}_0^{mn}$. The total Melnikov integral which estimates the splitting from all the screening currents and the vacuum field is

$$\tilde{M}_n = \tilde{M}_{n,\text{vac}} + \sum_{(m,n)\in S} I^\text{screen}_{mn} = \tilde{M}_0^{mn}$$

The ratio of the footprint extent measured in terms of $\psi$ for the screened vs. the vacuum field is given by $|\tilde{M}_n|/|\tilde{M}_{n,\text{vac}}|$. Table 1 lists this value for the COMPASS and JET cases as a function of the choice of screening currents.

In both cases a significant reduction of the footprints is predicted by the Melnikov integral when four screening currents are considered. To confirm this we plotted the stable manifold and a laminar plot around the inner strike point for COMPASS for the screened field and the stable manifold for the vacuum field for comparison (Figure 1, right plot). The stable manifold forms the boundary of the footprints as expected and indeed shows a clear reduction in comparison to the vacuum stable manifold. The difference of $\psi$ between the footprint tip and the base is $1.13 \times 10^{-4} \text{Tm}^2$, while the Melnikov integral method predicts $9.3 \times 10^{-5} \text{Tm}^2$. Similar result was found for the JET case (laminar plots are shown in [5]): the actual difference is $0.0023 \text{Tm}^2$, while the Melnikov integral method predicts $0.0026 \text{Tm}^2$.

**DISCUSSION AND CONCLUSIONS**

We developed a model of the plasma response currents on resonant surfaces and the resulting field, based on the realistic geometry of poloidally diverted tokamak plasmas and thus appropriate for the region near the separatrix, which is crucial for the ELM mitigation by external perturbations and also for the impact of perturbations on the divertor strike points (strike point splitting). To compute the screening currents we used the assumption of complete screening of resonant modes in the edge region, because we do not simulate the plasma response self-consistently. This is justified by earlier results indicating that the resonant modes of the perturbation will be suppressed by strong gradients in the pedestal region.

The resulting screened field was used to model magnetic footprints on the divertor by tracing the field lines. For two example cases (single-null equilibria of COMPASS and JET with $n = 2$ perturbations) we have shown notable differences in comparison with the vacuum field. The screening significantly reduces the spiralling patterns of field lines coming from inside the plasma. The reduction of footprints can be efficiently estimated by the Melnikov integral method. The spirals are shortened along their axis, the position of the axis is not affected. Reducing the coil current in a vacuum model has a similar impact. Comparison with experimental observation of strike point splitting could be thus used for validating the starting assumption about the screening.

The results can be qualitatively understood from the fact that the screening field is mostly localized at the LFS, just as the field of the coils. It can be shown that for a LFS-localized perturbation field the Melnikov integral which estimates the splitting is linked to the values of the Melnikov-like
function on the resonant surfaces which is proportional to the resonant modes of the perturbation [18]. Thus eliminating the resonant modes shall also mostly eliminate the Melnikov integral and splitting. It shall be noted that for a single screening surface the field is not at all localized so this reasoning does not apply. Indeed, we have seen that a single screening surface reduces the Melnikov integral only weakly.

The assumption of strong screening is supported by models which use a cylindrical geometry, but in the edge region realistic geometry is important. Our approach is thus not fully self-consistent. We have quantified the difference between a cylindrical and a realistic geometry by calculating the screening currents only with the diagonal components of the coupling matrix, as supposed by the cylindrical approach. The resulting currents are significantly lower than the currents obtained using the full matrix. We argue that the cylindrical models will underestimate the perturbation amplitude needed for penetration. Development of MHD models in realistic geometry which would be able to treat realistic plasma parameters and important effect such as diamagnetic screening will be required for a truly self-consistent treatment.

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Table 1: One mode Melnikov integral of the screened perturbation normalized to the Melnikov integral of the vacuum perturbation, for different choices of the screening currents.
Figure 1: Laminar plot of the connection length on the divertor of COMPASS near the HFS strike point with a vacuum (left) and screened (right) perturbation field. White line: The stable manifold of the vacuum perturbation field. Black line: the stable manifold of the screened perturbation field.

Figure 2: Poloidal dependence of the real part of the $n=2$ component of the relative perturbation field $b_1^n$ at the $m = 8$ resonant surface (for JET, Pulse No: 79729) and $m=7$ (for COMPASS). Fields of one screening current (full lines) and four screening currents (dashed lines) are shown.