Physics Basis and Validation of MMM7.1 Anomalous Transport Module
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Anomalous Transport Module

T. Raq¹, A.H. Kritz¹, R.V. Budny², I. Voitsekhovitch³, A.Y. Pankin⁴
and JET EFDA contributors*

JET-EFDA, Culham Science Centre, OX14 3DB, Abingdon, UK

¹Lehigh University, Bethlehem, PA, USA
²Princeton, Plasma Physics Laboratory, Princeton New Jersey 08543, USA
³JET-EFDA, Culham Science Centre, OX14 3DB, Abingdon, OXON, UK
⁴Tech-X Corporation, CO, USA

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ABSTRACT:
The Multi-Mode anomalous transport module version 7.1 (MMM7.1), recently installed in the PTRANSP code, is used to compute thermal, particle and toroidal angular momentum transport. In this study, the Weiland component of the theory based MMM7.1 is derived and simulations of DIII-D and JET tokamak discharges are carried out using the PTRANSP predictive integrated modelling code with boundary conditions taken from evolving experimental data. The time evolution of temperature profiles predicted using the MMM7.1 transport module are compared with corresponding data from DIII-D and JET tokamak discharges. The discharges simulated include L-mode and H-mode plasmas with co- and counter-rotations.

1. INTRODUCTION
Various confinement regimes of tokamak plasmas such as Low confinement mode (L-mode), improved L-mode (I-mode), High confinement mode (H-mode), Supershots, and internal transport barriers have been identified by experimentalists. The goal is to develop a theory based anomalous transport module to understand the interaction between physical processes that influence transport in these different modes of tokamak operation.

The MMM7.1 anomalous transport module consists of a combination of contributions from different transport theories. The MMM7.1 module is recently installed in PTRANSP, the TRANSP analysis code extended to carry out predictive integrated modelling simulations, and is used to compute thermal, particle and toroidal angular momentum transport. The MMM7.1 is documented and organized as a stand-alone module, which fully complies with the National Transport Code Collaboration (NTCC) standards [1] and is now available in the NTCC Module Library. The MMM7.1 has a single clearly defined interface, which facilitates porting the module to whole device modelling codes.

The purpose of the MMM7.1 anomalous transport model is to compute the evolution of electron and ion temperature profiles, particle (electron, hydrogenic and impurity) density profiles, and angular velocity profiles. The MMM7.1 module includes transport driven by instabilities associated with the ITG, TEM, KBM, MHD modes (Weiland module by J. Weiland and his group at Chalmers University in Sweden [2]), DRIBM modes (Raq module for Drift-resistive-inertial Ballooning Modes [3]) and ETG modes (Horton model for anomalous transport driven by Electron Temperature Gradient (ETG) modes [4] with the Jenko threshold [5]).

The combination of modes in MMM7.1 is necessary in order to include the variety of different physical phenomena that affect the plasma transport. These components of the MMM7.1 model provide contributions to transport in the different regions of plasma discharge. It has been found in the DIII-D and JET L-mode simulations carried out using MMM7.1, that the DRIBM contributes to the anomalous transport primarily near the edge of the plasma where the transport associated with ITG and TE modes are diminishing as a function of radius, while neoclassical ion thermal transport contributes mainly near the centre of the discharge.
2. COMPONENTS OF MMM7.1

2.1 WEILAND MODEL

The fundamental equations used in this model are the reduced Braginskii multi-fluid equations including charged-particle drifts. It is assumed that all of the perturbed quantities are proportional to \( \exp(\mathbf{k} \cdot \mathbf{r} - \mathbf{i} \omega t) \) where \( \mathbf{k} \) and \( \omega \) are the wave vector and the frequency. The Weiland transport model is a reactive fluid model that includes the fluid resonance in the energy equation. By reactive it is meant that dissipation is not involved in the closure. The principle of closure is that all moments with external sources in the experiment are included. A non-Markovian mixing length rule is used in order to separate the effects of ion modes on electron transport and electron modes on ion transport. This rule is used because the Doppler shifts due to the respective magnetic drift frequencies are included in the dependence on the real frequency. With this choice, the transport from all instabilities on all channels can be self consistently included by the addition of each contribution. The Weiland model include the effects of collisions, fast ion dilution, impurity dilution, non-circular flux surfaces, finite beta, and the Shafranov shift. The new Weiland model for drift modes has gone through a significant evolution from the previous model used as a component in the MMM95 transport model [6] to the new Weiland component in the MMM7.1 transport model. The new ITG/TE model in MMM7.1 module more accurately computes finite beta effects and the suppression of transport at low and reverse magnetic shear. The MMM7.1 module is further improved by making better approximations to the structure of the eigenfunctions along field lines in order to include the effects of non-circular flux surfaces, finite beta, and Shafranov shift.

2.1.1 FLUID ION EQUATIONS

For ions with density \( n_i \), the equation of continuity is

\[
(-\omega + \omega_{Di}) \dot{n}_i + \omega_{Di} \dot{T}_i + \left[ (\omega_{Di} - \omega_i)Z_i T_e/T_i - k_{gi}^2\rho_i^2 (\omega - \omega_i(1 + \eta_i)) \right] \phi + k_{||} c_s \dot{v}_{||i} = 0. \tag{1}
\]

The parallel ion motion \( \dot{v}_{||i} \) is determined by the parallel ion momentum equation driven by electromagnetic forces as well as by the ion pressure gradient and momentum transfer along the field lines. Consequently, the equation for \( \dot{v}_{||i} \) takes the following form

\[
\omega \dot{v}_{||i} = Z_i k_{||} c_s \left[ \phi + \left( \frac{\omega_i(1 + \eta_i) - \omega}{c k_{||}} \right) \tilde{A}_{||} \right] + \frac{T_i}{T_e} c_s k_{||} (\hat{n}_i + \hat{T}_i). \tag{2}
\]

The ion energy balance equation is

\[
(-\omega + \frac{5}{3} \omega_{Di}) \dot{T}_i + \frac{2}{3} \omega \dot{n}_i + \omega_{Di} \frac{g_{ni}}{2} (\eta_i - \frac{2}{3}) \phi = 0, \tag{3}
\]

Here \( \hat{n}_i \equiv \text{\tilde{n}}_i/n_i, \hat{T}_i \equiv \text{\tilde{T}}_i/T_i, \hat{\phi} \equiv \text{\tilde{\phi}}/T_e \text{\tilde{A}}_{||} \equiv \text{\tilde{e}} \text{\tilde{A}}_{||}/T_e \), and \( \hat{v}_{||i} \equiv \text{\tilde{v}}_{||i}/c_s \) are dimensionless forms of the perturbation, where \( \text{\tilde{n}}_i, \text{\tilde{T}}_i, \text{\tilde{\phi}}, \text{\tilde{A}}_{||}, \text{\tilde{v}}_{||i} \) are the perturbed ion density, ion temperature, electrostatic
potential, parallel component of vector potential and parallel ion flow, respectively. The \( \omega_i \) is the ion diamagnetic and \( \omega_{Di} \) is the ion magnetic drift frequency. The \( \eta_i \) is the ratio of ion temperature gradient to the density gradient, \( c_s \) is the sound speed, \( \rho_{si} \) is the ion Larmor radius and \( g_{ni} \) is the normalized density gradient.

2.1.2 ELECTRON EQUATIONS

The electrons can be divided into two classes: trapped (with density \( n_e \) and fraction \( f = n_{et}/n_e \)) and free (with density \( n_{ef} \) and fraction \( 1-f \)) with \( n_e = n_{et} + n_{ef} \). The electron density \( n_e \) is related to the density of hydrogenic ions \( n_{H} \); impurity ions \( n_{Z} = fZn_{e} \) and superthermal hydrogenic ions \( n_{s} = f_{s}n_{e} \), through charge neutrality \( n_{e} = n_{H} + Zn_{Z} + n_{s} \). The normalized perturbed densities (such as \( \hat{n}_e = \tilde{n}_e/n_e \)) are then related by

\[
\hat{n}_e = f_t \hat{n}_{et} + (1 - f_t) \hat{n}_{ef} = (1 - Zf_{Z} - f_{s}) \hat{n}_{H} + Zf_{Z} \hat{n}_{Z},
\]

assuming that superthermal ions do not take part in the perturbation, i.e., \( n_e = 0 \):

The equation for trapped electron continuity is derived from a kinetic equation including a Bhatnagar-Gross-Krook (BGK) collision term for trapped particles and in the limit \( \omega << \Omega e \) and ignoring electron finite Larmor radius effects \cite{7}

\[
(\omega - \omega_{De} + i\nu \omega_{De}) \hat{n}_{et} + (\omega_{De} - \omega_{e}) \hat{\phi} = \omega_{De} \hat{T}_{et} + i \nu \omega_{De} F,
\]

where \( \hat{n}_{et} = \tilde{n}_e/n_e \), \( \hat{T}_{et} = \tilde{T}_{et}/T_{et} \), \( F \equiv \Gamma \phi, \Gamma = 1 + g_{fe}/(\omega/\omega_{De} + i\nu - 1) \); \( g_{fe} \) is the normalized electron temperature gradient and \( \nu = (\nu_{e}/\omega_{De})R/r \) with \( \nu_{e} \) is the electron collision frequency.

The trapped electron temperature is determined by the following energy equation

\[
\left( \omega - \frac{5}{3} \omega_{De} \right) \hat{T}_{et} = \left[ \omega_{e} \left( \eta_{e} - \frac{2}{3} \right) + i\nu_{th} \right] \hat{\phi} - \frac{5}{3} \nu_{th} F + \frac{2}{3} (\omega + i\nu_{th}) \hat{n}_{et}.
\]

The free electron continuity equation can be rewritten as

\[
(\omega - \omega_{De}) \hat{n}_{ef} = (\omega_{e} - \omega_{De}) \phi - \frac{k_{th}}{e n_0 B r} \frac{\partial J_{||e}}{\partial r} A_{||} + \omega_{De} \hat{T}_{ef} + k_{||} \tilde{v}_{||e},
\]

where \( n_{et} = n_{ef}/n_{ef}, v_{||e} = v_{||}/v_{||o}, \) and \( T_{et} = T_{ef}/T_{ef} \). A relation between the perturbed free (circulating) electron density \( n_{ef} \) and the perturbed electric and magnetic potentials can be obtained from the momentum equation for free electrons parallel to the unperturbed magnetic field. Assuming electron velocity parallel to magnetic field is much greater than the parallel ion velocity and assuming \( v_{e} >> \omega \); the perturbed parallel electron motion gives

\[
\hat{n}_{ef} = \hat{\phi} - \hat{T}_{ef} + \frac{(1 + \eta_{e}) \omega_{e} - \omega}{ck_{||}} A_{||} - \frac{\tilde{v}_{||e}}{ik_{||} D_{e}}.
\]
where \( D_e = 2T_e/m_v \). The free electron temperature is assumed to be isothermal as

\[ T_{ef} = \eta e^{\frac{\omega_s e}{c k \|}} T_e. \quad (9) \]

Through the use of Equations (7), (8), (9) and the use of the toroidal component of Ampere’s law, \( J_{\|0} = \frac{1}{\mu_0} \frac{1}{r} \frac{\partial}{\partial r} (r B_\theta) \); the relation between normalized electrostatic potential (\( \hat{\phi} \)) and vector potential (\( \hat{A}_\| \)) can be obtained

\[
(\omega - \omega_{ce}) \hat{\phi} = \left[ \frac{I}{c k \|} \omega (\omega - \omega_{ce}) + \omega_{De} (\omega_{ceT} - \omega) - k_\| k_\theta \rho_s^2 k' \| \hat{A}_\| \right] - k_\|^2 \rho_s^2 k_\| \|^{\|} \hat{A}_\| - i \omega_{De} \frac{m_e R}{m_i c_s} \frac{k_\|^2}{k_\| \|} \hat{A}_\| \left( 1 - \frac{\omega}{\omega_{De}} \right)
\]

\[ (10) \]

where \( \omega_{ceT} \) is the diamagnetic drift frequency with temperature gradient, \( v_A^2 = B^2/nm_i \) is the Alfvén speed, and \( \beta_e = nT_e/B^2 \), is the ratio of electron pressure to magnetic pressure. The third term in the right side of the Eq.(10) is not included in the simulations reported here. The impurity equations, not described here, are analogous to main ion continuity Eq.(1), momentum Eq.(2) and energy equations Eq.(3).

An effect of the extended eigenfunctions is illustrated by the following partial derivation. Note that the magnitude of the ion magnetic drift frequency varies strongly around each magnetic surface

\[ \omega_{Bij} = \frac{-2k_B T_i}{Z_j eBR} g(\theta) \quad (11) \]

where, for Shafranov-shifted circular magnetic surfaces,

\[ g(\theta) = \cos \theta + s \theta \sin \theta + \alpha_m \sin^2 \theta. \quad (12) \]

Here, \( s \) is the magnetic shear and

\[ \alpha_m = \frac{2 \mu_0 R q^2 dp}{B^2 dr} \quad (13) \]

is proportional to the Shafranov shift of the magnetic surfaces relative to one another. Consider eigenfunctions that are extended along each magnetic field line with a poloidal angle dependence that is proportional to \( \exp(-\alpha \theta) \). The coefficient \( \alpha \), which is determined by asymptotically matching the eigenfunction solution at large poloidal angles, \( \theta \rightarrow \infty \), is given by

\[ \alpha = |\text{Re}(\omega)| k_\| R \rho_s^2 |s| g/\omega_{De}, \quad (14) \]

where \( \omega \) is the eigenvalue (frequency and growth rate) associated with the mode. The flux-surface
average of the geometric factor, \( g(\theta) \), times the eigenfunction has the form

\[
\langle g(\theta) \exp(-\alpha \theta^2) \rangle = \left( 1 + \frac{s}{4\Re(\alpha)} \right) \exp \left( -\frac{1}{8\Re(\alpha)} \right) + \frac{\alpha_m}{2} \left[ 1 - \exp \left( -\frac{1}{8\Re(\alpha)} \right) \right] \tag{15}
\]

The coefficient \( \theta \) and, therefore, the eigenvalue \( \omega \), appears in the denominator and exponential function in this flux-surface average. Since this kind of flux-surface average appears in several of the eigenvalue equations in the Weiland model, it can be seen that the eigenvalue equations are no longer linear in the new Weiland model. Scalings of the correlation length of drift wave turbulence with magnetic \( q \), shear, elongation, and temperature ratio have been introduced into the new Weiland drift wave transport model.

### 2.2 DRIFT-RESISTIVE-INERTIAL BALLOONING MODEL

The Raq DRIBM model [3] is based on a unified theory for resistive and electron inertial ballooning modes using a two-fluid model. The DRIBM model consists of six coupled equation, which are derived from Ohms law, vorticity, continuity, total parallel momentum, and electron and ion energy equations. This model includes electron inertia, electromagnetic perturbations, diamagnetic effects, parallel ion dynamics, transverse particle diffusion, and perpendicular gyroviscous stress terms. The DRIBM model takes into account the effects of electron and ion temperature gradients, temperature perturbations, and parallel conductivity. The DRIBM model describes pressure driven modes that are driven around the outboard edge of toroidal plasmas, where the magnetic field lines are concave to the plasma. The DRIBM model replaces the semi-empirical model for resistive ballooning modes [8] and kinetic ballooning model [6] used in the MMM95 transport model.

### 2.3 THE HORTON ETG MODEL

The Horton ETG model results from a generalization of a hydrodynamic theory for short wavelength ETG turbulence with electromagnetic effects included [4]. The calibration of this model was carried out using data from fast wave electron heated discharges with hot electrons in Tore Supra experiments. The ETG mode is considered as a lower hybrid drift mode in the toroidal direction, driven unstable by charge separation caused by the combined effects of the VB and curvature on electron drift in the presence of an electron temperature gradient. Two space scale regimes are involved in the Horton ETG model depending on the wavelength of the ETG modes, which generally determines the electron thermal transport in different plasma regions. While the long wavelength regime is neutrally stable, the short wavelength regime can drive electrostatic turbulence. The coupling of short wavelength electrostatic fluctuations with the longer space scale can also drive secondary electromagnetic turbulence.

### 3. SIMULATION RESULTS

Experimental data from JET and DIII-D discharges are considered in this paper. Simulations of JET
and DIII-D tokamak discharges are carried out using the PTRANSP predictive integrated modelling code with time evolved boundary conditions. The discharges simulated in the validation study of MMM7.1 include JET L-mode (Pulse No: 79575), DIIID H-mode plasmas with co-rotation (82205) and with counter rotation (Pulse No: 99251) [9]. The time evolution of temperature and current density profiles predicted using the MMM7.1 transport module are compared with corresponding experimental data at the diagnostic time. Evolution of plasma discharges utilizes experimental boundary and experimental initial conditions. The toroidal frequency, the electron density and the effective charge of plasma are obtained from experimental data. The ion density is determined by employing quasi-neutrality. The NUBEAM Monte Carlo module for NBI is used to calculate heating power deposition and current drive. Electron and ion thermal transport is computed using a combination of neoclassical and anomalous transport models. The MMM7.1 is used to compute anomalous transport from magnetic axis to plasma edge in L-mode discharges, and from magnetic axis to top of pedestal in H-mode discharges.

The comparison of the simulated and experimental profiles for JET Pulse No: 79575 at 17 seconds is shown in figure 1. The experimental profiles are represented by dashed lines, and the profiles predicted using the MMM7.1 transport model are represented by solid black lines.

Pulse No: 82205 is part of a DIII-D GA scan designed to have the same plasma shape as well as the same plasma $\beta$, collisionality, and safety factor as ITER [9]. Pulse No: 99251 has been used in $\rho^*$ and plasma rotation scans. The directions of the toroidal field and plasma current in this discharge were in the reversed $I_p$ DIII-D directions. This discharge is a close match for the co-$I_p$ rotation Pulse No: 82205. The simulated temperature for counter rotation Pulse No: 99251 is compared with experimental data in figure 2 and for co-$I_p$ rotation Pulse No: 82205 in figure 3. The boundary condition in these simulations is set at $\rho=0.8$. The toroidal rotation profile is taken from experimental data and not predicted. Further validation studies will be carried out comparing MMM7.1 simulation results with more experimental data. Density and rotation profiles will also be simulated along with electron and ion temperatures profiles.

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**REFERENCES**


Figure 1: Simulated profiles solid curves and experimental data dashed curves for ion temperature (upper panel), electron temperature (lower panel), as function of normalized radius for JET Pulse No: 79575 L-mode discharge at 17.0sec.
Figure 2: Simulated profiles solid curves and experimental data dashed curves for ion temperature (upper panel), electron temperature (lower panel), as function of normalized radius for DIII-D 99251 H-mode discharge at 2.2 sec. It is a counter beam current discharge with D-NBI injected from 1.5 sec to 4.0 sec.

Figure 3: Simulated profiles solid curves and experimental data dashed curves for ion temperature (upper panel), electron temperature (lower panel), as function of normalized radius for DIII-D 99251 H-mode discharge at 2.5 sec. It is a co-beam current discharge and closely match to the counter-Ip rotation Pulse No: 92251 shown in figure 2.