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A Dimensionless Criterion for Characterising Internal Transport Barriers in JET

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ABSTRACT

An apparent magnetic-field-dependent power threshold for the emergence of Internal Transport Barriers (ITB) in JET is shown to be embedded in the ratio of the ion gyro-radius to the local gradient scale length. The existence of ITB’s can be inferred in regions of space-time where this dimensionless parameter exceeds some critical value. The underlying physics leading to its theoretical relevance as a local indicator of a bifurcated plasma state is the stabilisation of turbulence by the E×B shear. Large database analysis and real-time plasma control are envisaged as attractive applications.

Several recent experiments realised in various tokamaks have demonstrated the existence of high performance regimes in which a so-called internal transport barrier (ITB) appears [1]. An ITB is defined as a plasma region where anomalous transport is eliminated or strongly reduced, and, because of the important role of the local magnetic and flow shears for the emergence of such barriers, discharges with ITB’s have been refered to on JET as the “optimised shear” (OS) discharges [2]. The analysis of the JET OS database frequently requires the identification of some typical ITB features like the onset time, location, collapse time and dynamics of the barrier.

An intuitive method employed so far is often to locate a visible “break” on the temperature profiles or, alternatively, one can resort to the computation of the thermal diffusivity profiles and look for a region of reduced transport. These somewhat subjective or rather cumbersome procedures for assessing the existence and location of ITB’s are not efficient for an extensive database study, and are inadequate for an active control of their evolution as proposed for instance in [3]. Hence, beyond the existence of an apparent magnetic-field-dependent power threshold governing the emergence of ITB’s [2], there is a real interest in developing some physical and practical criterion which could be used routinely for an objective characterisation of the ITB features and possibly for controlling their dynamics in real time.

A local ITB criterion serving these purposes is proposed in this report. We first describe the underlying physics, through a simple dimensional analysis involving the main ingredients of microturbulence theory in magnetised plasmas, then discuss how the measurements uncertainties can be taken into account, and finally show the relevance of the proposed ITB criterion on the JET experimental database.

The physical mechanisms of barrier formation have not yet been completely identified, but drift waves are thought to be the principal vector of microturbulence when the plasma is driven far from thermodynamic equilibrium, and their stabilisation is likely to be the cause of a transport...
reduction leading to the emergence of ITB’s [4, 5]. Despite a variety of possible unstable modes, such as the Ion Temperature Gradient (ITG) or the Trapped Electron Modes (TEM), a fundamental characteristic length arises from their dispersion relations: the ion Larmor radius at the sound speed, \( r_s = c_s/w_ci \), where \( c_s \) is the ion sound speed, and \( w_ci \) the ion cyclotron pulsation. Several studies have been carried out on various tokamaks to find how the thermal diffusivities scale with respect to \( r_s \) (or more exactly to the normalised Larmor radius \( r^* = r_s/a \), \( a \) the minor radius), and have given rise to the so-called “Bohm” or “gyro-Bohm” scalings (e.g. [6]). When transport barriers appear, local gradient scale lengths become much shorter than the plasma size and, for a local analysis, one should indeed normalise the drift wave scale length \( r_s \) to the local temperature gradient scale length, e.g. \( L_{Ti} = -T_i/(\partial T_i/\partial R) \) or \( L_{Te} = -T_e/(\partial T_e/\partial R) \) where \( T_i \) (resp. \( T_e \)) is the ion (resp. electron) temperature and \( R \) is the plasma major radius on the equatorial plane. We therefore define the local dimensionless Larmor radius, \( r^*_T \), as \( r^*_T = r_s/L_T \) and consider either \( r^*_{Ti} \) or \( r^*_{Te} \).

A possible mechanism for the stabilisation of the ITG and TEM modes in tokamaks combines the \( E \times B \) rotational flow and the magnetic shear effects, as was found by several authors through the extensive use of computer codes [7-12]. Shear flows tend to destroy the global character of the toroidal modes, through the differential rotation of the coupled local modes, and can also lead to the full stabilisation of the modes. Non-linear simulations of ITG modes showed that this occurs when the \( E \times B \) shear rate, \( g_E \), exceeds the maximum linear growth rate of the local modes, \( g_{max} \) [8, 9].

Most of the stability analysis of transport barriers rely on this relation, which is the starting point of our analysis. We use here the cylindrical version of the \( E \times B \) shear rate, \( g_E = d(E_r/B_T)/dr \), but using the more accurate toroidal definition [13] does not change the scaling. The radial electric field in a tokamak is related to the ion pressure gradient and velocity through the relation

\[
E_r = (z_i e n_i)^{-1} (dp_i/dr) - V_{\theta i} B_{\theta} + V_{\phi i} B_{\phi}
\]  

If the poloidal velocity \( V_{\theta i} \) is assumed to be given by the neoclassical theory, the corresponding term in (1) scales as the pressure gradient term. Then, the contributions of the pressure and poloidal velocity in the shear rate, which correspond to the “diamagnetic part” of the rotational shear flow, scale as

\[
\gamma_{E,dia} = (c_s/L_T) r^*_T
\]  

where we have used the same length, \( L_T \), to dimensionally estimate the gradients and their radial derivative. Now, the growth rate of a drift micro-instability scales as

\[
\gamma_{max} = (c_s/L_T) r^*_IB \Lambda_{T,s,q,B,V^*,...}
\]
where the function $\rho_{ITB}^*$ describes the stability of a specific mode. For a class of profiles at given $\beta$ and collisionality, $v^*$, this function should depend mainly on the gradient length, $\Lambda_T = L_T/R$, and on the details of the safety factor profile, $q(r)$, and magnetic shear, $s$. In $\gamma_E$, the toroidal velocity shear term, $\gamma_{E,\phi}$, scales as $M_0 c_s / L_T$, where $M_0$ is the Mach number, and could be subtracted from the growth rate $\gamma_{\text{max}}$ when comparing $\gamma_E$ and $\gamma_{\text{max}}$.

The key point here is that the stabilisation criterion $\gamma_E > \gamma_{\text{max}}$ (or $\gamma_{E,\text{dia}} > \gamma_{\text{max}} - \gamma_E$) can be recast in the form $\rho_T^* > \rho_{ITB}^*$. In the simplest case of self-similar discharges, or if the parameter dependences in $r_{ITB}^*$ are weak, this criterion leads to a critical value of $\rho_T^*$, whereas, if the gradient length is close to the instability threshold value $L_T^c$, $\rho_{ITB}^*$ is expected to be proportional to $1 - L_T / L_T^c$. This physics can be implemented into a transport model by using a mixing length argument, $\chi_{\text{turb}} = g_{\text{eff}} L_c^2$, where $g_{\text{eff}}$ is an effective growth rate ($g_{\text{eff}} = g_{\text{max}} - g_E$), and $L_c$ is the turbulence correlation length. Turbulence simulations show that $L_c$ is proportional to the gyroradius $r_s$ in most cases [10, 11] so that the resulting expression for the heat diffusivity reads

$$\chi_{\text{turb}} = \rho_s c_s F(\Lambda_T, s, ..., \rho_T^*, \rho_{ITB}^* - \rho_T^*)$$

(4)

where the dimensionless function $F$ measures the stiffness of the model. At large $\rho_T^*$, the gyro-Bohm scaling of the heat diffusivity is thus weakened – and eventually broken – by the diamagnetic flow shear stabilisation. We now conjecture that, if $\rho_T^*$ is a relevant parameter, reducing locally the temperature gradient scale length (through intense heating) can have a stabilising effect similar to that of increasing the normalised Larmor radius ($\rho^* = \rho_s / a$) at constant $L_T / R$, as is generally done in the numerical simulations. A typical set of S-shaped bifurcation curves [14] can be obtained by plotting the heat flux as a function of $\rho_T^*$ since (Figure 1):

$$\Gamma_{\text{turb}} = n T c_s F(\Lambda_T, s, ..., \rho_T^*, \rho_{ITB}^* - \rho_T^*)$$

(5)

and therefore increasing the power flux could eventually result in a transition between the gyro-Bohm branch of the curves and the ITB branch (e.g. neoclassical transport) although $\rho_T^*$ is not the only parameter which varies and the exact multidimensional dynamics involves other independent parameters (e.g. $\Lambda_T$, $s$, $\beta$, ...) as well as other transport coefficients (density, momentum).

We can then test whether an ITB existence criterion could possibly be expressed according to the local value of $\rho_T^*$ such as:

$$\rho_T^*(R, t) > \rho_{ITB}^* \iff \text{an ITB exists at radius R and time t}$$

(6)

A critical value $\rho_{ITB}^*$, if it does exist, can be evaluated experimentally by comparing discharges with and without ITB’s, and with different dimensional parameters such as toroidal field, main ion species, plasma current, etc…
The criterion was tested on the JET OS database for various types of plasmas. The measurements required for computing $\rho_T^*(R,t)$ are the electron and/or the ion temperature profiles, and the toroidal magnetic field, e.g. on the magnetic axis. Unavoidably all these data are corrupted by uncertainties quantified by their associated standard deviations. Therefore the binary problem of deciding whether there is an ITB or not can also be answered in terms of a confidence factor for the criterion (6) to be fulfilled consistently with the measurement uncertainties. The electron temperature profiles were measured by the ECE heterodyne radiometer which offers an excellent resolution (0.5 ms/2 cm) and an accuracy of about 5% whereas the ion temperature profiles were obtained by charge exchange spectroscopy with a much lower resolution (50ms/10cm) and an accuracy of about 5%. The term $L_T^{-1} = -\partial \ln T / \partial R$ is computed as the logarithmic radial derivative of the temperature on the equatorial plane by applying a three-point difference formula. The measurement uncertainties are combined with the standard propagation of errors to yield the relevant standard deviations for $\rho_T^*, \sigma_{\rho_T^*}$.

Then assuming a normal distribution,

$$p_{\rho_T^*, \sigma_{\rho_T^*}}(x) = \frac{1}{\sqrt{2\pi} \sigma_{\rho_T^*}} \exp \left( \frac{(x - \rho_T^*)^2}{2 \sigma_{\rho_T^*}^2} \right)$$  \hspace{1cm} (7)

the probability that $\rho_T^*$ exceeds the critical value is given by

$$\Phi_{\text{ITB}}(R,t) = \Phi[\rho_T^*(R,t) \geq \rho_{\text{ITB}}^*] = \int_{\rho_{\text{ITB}}^*}^{\infty} p_{\rho_T^*, \sigma_{\rho_T^*}}(x) dx$$  \hspace{1cm} (8)
This figure of merit will be referred to as a confidence factor for identifying an ITB at a given time and radius. An attractive representation of the results is obtained by plotting contours of either $\rho_T^*$ or $\varphi_{\text{ITB}}$ in the $(t, R)$ plane. The constant $-\rho_T^*$ contours are plotted only for $\rho_T^* > \rho_{\text{ITB}}^*$ and the constant $-\varphi_{\text{ITB}}$ contours are plotted only for $\varphi_{\text{ITB}} > 50\%$ ($\varphi_{\text{ITB}} = 50\%$ is obtained when the expectation value of $\rho_T^*$ equals $\rho_{\text{ITB}}^*$). Figure 2 shows an example of such a graph for a discharge where the ITB dynamics is rich of events. It can be seen that all the relevant information such as onset time, collapse times, as well as the expansion, contraction and width of the barrier appear explicitly for a low computational cost. The normalised radius, $\rho = (\Phi/\Phi_{\text{max}})^{1/2}$ with $\Phi$ the toroidal flux through a poloidal section and $\Phi_{\text{max}}$ its maximal value, has been used here instead of $R$ to label flux surfaces.

The critical value $\rho_{\text{ITB}}^*$ was chosen from a discharge with a perfectly visible barrier whose emergence time was well defined and which was used as a reference. It was thus found that $\rho_{\text{ITB}}^* = 1.4 \times 10^{-2}$ would match both the emergence time and radial evolution of the barrier satisfactorily. Pulses obtained with almost the same operating conditions but which either exhibit an ITB or not (as seen from the neutron yield) were also compared and successfully distinguished by the $\rho_{\text{T}}^* > 1.4 \times 10^{-2}$ criterion. ITB’s observed on the ion temperature profile can also be detected according to $\rho_{\text{T}}^*$ despite the smaller resolution of the $T_i$ measurements (Figure 3). Quite interestingly, the same $\rho_{\text{ITB}}^*$ value is again in good agreement with the information one can get independently from the detailed analysis of the discharges. Some plasmas also exhibit a double internal barrier such as #49680 [15]. The double barrier was observed on the electron temperature (Figure 4) and the $\rho_T^*$ criterion was again in good agreement with the observations. It must be noted that after a barrier has formed, $\rho_T^*$ is not limited to $\rho_{\text{ITB}}^*$, and that various degrees of temperature gradient strengths and barrier widths develop, as also observed on other tokamaks.

In order to evaluate its reliability for detecting the presence and evolution of ITB’s, our criterion was tested on many discharges from the JET OS database with various experimental conditions. For this purpose 116 deuterium pulses were selected with toroidal magnetic fields varying from 1.8 to 4 T, plasma currents from 1.6 to 3.6 MA (safety factors from 3.3 to 4.3), central densities from 2 to $5.5 \times 10^{19}$ m$^{-3}$, NBI powers from 4.8 to 18.7 MW and ICRH powers from 0 to 8.7 MW. Among them, 84 discharges presented an ITB. Their emergence times were then evaluated by detailed analysis and are confronted with the $\rho_{\text{T}}^*$ criterion on Figure 5. Only five very weak barriers were not detected by the criterion whereas one detection was not assessed by a detailed analysis. Dependences of $\rho_{\text{ITB}}^*$ on other dimensionless parameters as suggested in (3) did not arise despite a variety of scenarios. This should perhaps stimulate further investigations in which these parameters could be varied in a more systematic way, but it constitutes the main result of the present analysis. It must be noted that mostly electron temperature barriers were considered here because of the good spatial and temporal resolutions of the ECE diagnostic.
In summary, a tool for characterising the internal transport barriers on JET has been developed. It satisfies a certain number of properties such as the detection in both time and space of ITB’s independently of the experimental conditions, inclusion of the measurement uncertainties, ability to be applied to the electron or ion temperature profiles, and it is practical to use for the analysis of experimental data and the handling of large databases.

It was tested successfully on many JET discharges with only 5% of detection errors and with an ITB emergence time uncertainty of the order of 150 ms. The ρT * criterion provides a localisation of the ITB’s and displays their evolution for a low computational cost. Thus, it may find a number of applications on JET or other devices, such as the real-time control of ITB’s which could contribute to the development of advanced steady-state operation scenarios with reduced plasma turbulence in tokamaks.
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