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Triggering of neo-classical tearing modes by mode coupling

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Abstract

The formation of magnetic islands with mode numbers \((m>1, n=1)\) apparently triggered by sawtooth crashes has for a long time been a concern in JET discharges with auxiliary heating as it affects the peak performance, in terms of stored energy or of neutron yield (in DT experiments). The phenomenological association of the sawtooth crash with the onset of resistive modes is becoming particularly relevant in the context of Neoclassical Tearing Modes (NTM) theory that requires the formation of a “seed” island above a critical value. Relying on observations and on a critique of the assumed role of the collapse phase of the \((1,1)\) instability we propose as physical mechanism of destabilisation of the metastable NTM a forced reconnection process due to electro-dynamic mode-mode coupling. For specific frequency matching conditions a bifurcation pattern appears in the \(\Delta(W)\) plot. Modeling of a JET discharge shows that for perfectly frequency matching conditions the triggering of the \((m=3, n=2)\) NTM could be due to either two-mode toroidal coupling or three-mode non-linear coupling. The observed mode frequencies are consistent with the non-linear coupling.

1-Introduction

In several experiments performed at JET, the peak performance, in terms of neutron yield and plasma stored energy, has been observed to deteriorate in coincidence with the onset of fast
rotating MHD modes appearing around the time of large sawtooth crashes [1-4]. These MHD modes with poloidal to toroidal mode number ratio larger than one, $m/n>1$, appear at moderate to high $\beta$ values in both Elm-free and ELMy H-mode plasmas. The danger these MHD modes pose to future reactor plasmas was seen in the JET DT experiments, where the growth of $n=3$ and $n=4$ modes following a sawtooth crash were clearly associated with saturation of the fusion yield [1]. In recent JET campaigns, the triggering of magnetic islands at different $\beta$ values and their effect on confinement were extensively studied in ELMy H-mode plasmas designed to study the onset of Neo-classical Tearing Modes (NTM) [2-4].

The correlation between sawteeth activity and NTMs has been clearly demonstrated in JET experiments where reduction of the sawtooth period by ICRH techniques delayed the onset of NTMs [5]. However, the mechanism for triggering MHD modes around a sawtooth crash is still an open problem of tokamak physics. Here we consider the possibility that the sawtooth precursor, rather than the crash may make those modes unstable.

Inspection of high resolution magnetic data shows that in many discharges the onset of modes seen to grow after a sawtooth crash, actually occurs before the crash. In 1999, systematic work on JET in discharges where NBI was carefully ramped up [3, 3a], indicated that the onset of ($m=3$, $n=2$) NTMs was mostly correlated with the peak in magnetic sawtooth precursor activity, rather than with the crash event itself. Thus the ($m=3$, $n=2$) NTM onset started before the sawtooth crash, coexisting with the sawtooh ($m=1$, $n=1$) precursor and its ($m=2$, $n=2$) and higher harmonics. Further investigation of the correlation between sawtooth crash times and the ($m=3$, $n=2$) NTM onset has been recently obtained for a large number of discharges [6] and confirms that in the majority of cases the ($m=3$, $n=2$) NTM starts before a sawtooth and in the presence of central MHD modes, such as sawtooth precursors or fishbones.

For a correct interpretation it seems reasonable to consider the nature of perturbations labeled as sawteeth. The most immediate aspect of sawteeth, namely the non-linear relaxation oscillations of temperature, are an effect and not a cause of an MHD instability with a fixed helical mode ($m=1$, $n=1$) and cannot be considered a reasonable trigger of a resistive magnetic perturbation with a different helicity. Concerning the ($m=1$, $n=1$) perturbation, in the linear stage the fluid displacement $\xi(=W/2)$ (e.g. of the magnetic axis) grows exponentially to the crash limit $F_q = |r_{q=1} - r_0|$ where
reconnection changes topology of the magnetic surfaces on a 100 µs timescale (a moon shaped island is formed at the crash). More realistically the displacement $\xi$ after an initial exponential growth enters a non-linear evolution stage which may end up in: a) a crash similar however to the linear model, b) an $m=1$ island with amplitude $W$ slowly evolving and rotating [7]. The direct association of the collapse phase of the $(m=1, n=1)$ reconnecting perturbation with the formation of a seed island of different helicity at a flux surface far from the $q=1$ mixing radius is physically unclear and needs to be framed in a more rational picture. It should be mentioned that there are experiments in [8] where the seeding of NTMs seems, more reasonably, to be due to conventional tearing modes with the same $(m, n)$ numbers.

If the crash itself is a doubtful direct triggering mechanism, the data on the other hand suggests that electro-dynamic mode coupling [9-11] may play an important role in the destabilisation of these modes [3,12]. In [3] it was proposed that the $(m=3, n=2)$ modes observed in JET discharges may be destabilized by toroidal coupling with the $(m=2, n=2)$ sawtooth precursor. Alternatively, one could envisage non-linear coupling between modes with different toroidal numbers, with the growing $(m=1, n=1)$ sawtooth precursor as the driving mode [12]. A key aspect of the coupling hypothesis is whether certain definite relations between modes’ frequencies are experimentally observed. Examination of the 1999 JET data showed that the $(m=2, n=2)$ precursor that was supposed to couple to the $(m=3, n=2)$ mode, did not lock to it [3, 3a]. This suggests either a very brief locking phase or some non-linear coupling mechanism. On the other hand, if we consider the $(m=4, n=3)$ mode which is often present at the $(m=3, n=2)$ onset, and a smoothly monotonic toroidal plasma rotation profile, we find that the difference between the $(m=4, n=3)$ and the $(m=1, n=1)$ mode frequencies provide a near match to the frequency of the $(m=3, n=2)$ mode. This indicates the possibility of non-linear coupling between those three modes.

Therefore we present a discussion in general terms of the conditions and the effectiveness of mode coupling in perturbing a marginally stable situation. We consider in particular the basic toroidal [9] and non-linear [10, 11] coupling mechanisms affecting marginal stability of NTMs. The theoretical argument is supported by referring to specific cases observed in the JET ELMy H-mode regime [3]. In those discharges the conditions are slowly ramped through the threshold for destabilisation.
of NTMs. Accordingly we formulate the problem of assessing the conditions for driving a reconnected island above threshold, by linear or non-linear coupling of modes.

2-Experimental observations and theoretical model

Without loss of generality, we concentrate on a study case, JET discharge No 47285, obtained in a campaign of study of NTMs (described in detail in [3a]) and characterized by a gradual stepping of the Neutral Beam Injection heating power. A \((m=3, n=2)\) starts to grow when the heating power is \(P_{\text{NBI}}=12\text{ MW}\) (Fig.1). The MHD modes observed around the time of the \((m=3, n=2)\) onset are shown in Figures 1 and 2. In Fig.(1) the time sequence of observed events is shown; in particular the second trace shows that the \(n=2\) (NTM) magnetic perturbation starts growing well before the crash of the \(n=1\) mode that corresponds to a sawtooth crash. The other traces show the behaviour of the \(D_\alpha\) emission, the central electron temperature and the neutral beam power waveform.

The spectrogram of the modes observed, shown in Fig.(2b), gives evidence that the onset of the \((m=3, n=2)\) mode, occurs clearly earlier than the sawtooth crash of the \((m=1, n=1)\) (at \(t=22.37s\)) and in presence of a \((m=4, n=3)\) mode. The \((m=4, n=3)\) mode appears in bursts, each burst ending with a reduction in the observed frequency. The \((m=3, n=2)\) mode is first observed at \(t=22.345s\) following the crash of the \((m=4, n=3)\) mode. It appears to be interrupted by an ELM, then it starts to grow irreversibly at \(t=22.36s\), as the \((m=4, n=3)\) mode re-starts to grow and the \((m=1, n=1)\) mode is reaching maximum amplitude. The \((m=4, n=3)\) crash is followed by a decrease in frequency, which transiently provided a nearly perfect matching condition between the rotation speed of these three modes (fig. 2c), satisfying \(\omega_{(1,1)} = \omega_{(4,3)} - \omega_{(3,2)}\), suggesting a mechanism of non-linear coupling that will be explained below.

Therefore the following sequence of events can be conjectured and discussed. Initially, the local \(\beta_\theta\) for the driven mode \((m=3, n=2)\) and a “passive” mode \((m=4,n=3)\) is raised by NBI marginally above \(\beta_{\text{crit}} = \rho_{\text{it}} \left( a^2 e^{-3/2} \right)^{1/2} \left( L_p L_q^{-1} g \right)^{1/2} \left| \epsilon \Delta \right|\) which is the critical value for onset of neo-classical tearing instability with \(r_s \Delta < 0\) and typical pressure gradient and shear lengths \(L_p = p/p'\) and \(L_q = q/q'\). However, the reconnected island width of the \((m=3, n=2)\) mode at \(t=22.35s\) is
below the threshold value $W_{\text{rec}} < W_{\text{thr}}$, therefore the nonlinear growth of the instability is prevented. When the driving mode ($m=1, n=1$) grows non-linearly to a sufficiently large amplitude, driven by its own free energy, the non-linear coupling with the other two provides a larger forced reconnection mechanism for the ($m=3, n=2$) mode, satisfying $W_{\text{rec}} \geq W_{\text{thr}}$.

A simple theoretical model of non-linear coupling of triplets of rotating magnetic islands was developed in Ref [10], as an extension of the large $R/a$ Rutherford theory of non-linear tearing modes in a limit of low $\beta$. This includes the mutual electro-dynamic and viscous coupling of a triplet of rotating magnetic islands of different helicities arranged to obtain a "resonant wave-number matching". This mechanism of coupling is different from that arising from the sideband poloidal harmonics generated by the toroidal geometry and involves modes with different toroidal numbers $n$.

In analogy with a three-wave interaction, coupling may arise among modes with mode numbers collectively labeled as $\tilde{k} \equiv (m, n), \tilde{k}' \equiv (m', n')$ and $(\tilde{k} - \tilde{k}') \equiv (m - m', n - n')$. Magnetic perturbations are represented by $\delta \mathbf{B}_{mn} = \nabla \times \left( \psi_{mn} \mathbf{b}_{mn} \right) = \delta \mathbf{B}_{mn}(r) \cdot \mathbf{e}^{i \left( \frac{m \theta - n}{R} z + \zeta_{mn} \right)} + \text{c.c}$ and are related to the magnetic island width $W_{mn} = \left( \psi_{mn} / h_{mn} \right)^{1/2}$ where $h_r = B_{\phi} r s q / 16 R q^2$. The island mechanics for a triplet of “wave-numbers” $k, k', k''$ fulfilling a matching condition $k'' = k - k'$ is described by coupled equations for $W_k(t)$ and the rotation frequency $\omega_k$.

Here, the role of coupling as a forced reconnection mechanism is investigated. The coupling model considered here, as well as that for toroidal coupling, cannot describe the early stage of the forced reconnection. However, here attention is drawn only to the non-linear stage, when some reconnection has already occurred and the mode frequency is already that of the driving mechanism (if one considers three-wave coupling and $(m - m', n - n')$ as the driven mode, then the driving frequency is $\omega_{(m,n)} - \omega_{(m',n')}$.  

From the experimental observations (data shown in Fig. 2), it is adequate to consider the mode frequency practically constant while the width evolves non-linearly as:

$$\frac{dW_k}{dt} = \Gamma_k(W_k, W_{k'}, W_{k''}, \Delta \phi) \quad (1)$$
Here \((k, k', k'') = (1,2,3)\) label in turn one mode of the coupled triplet and \(\Delta \phi\) is a phase difference between modes and in absence of coupling \(\Gamma_k^{(0)}(W < W_{thr}) \leq 0.\)

The basic principle of triggering of metastable NTMs by mode coupling is illustrated in figure 4, where the growth rate \(dW/dt\) dependence on \(W\) is shown for both the \(\chi_\perp/\chi_\parallel\) [13] and polarization current models [14], with and without coupling effects and with \(\beta_0 > \beta_{\theta, crit}\) so that the mode is metastable.

In the absence of mode coupling effects, the \(\chi_\perp/\chi_\parallel\) and polarization current models yield threshold island widths given by \(W_{thr}^{(1)}\) and \(W_{thr}^{(2)}\), respectively, with \(W_{thr}^{(2)} > W_{thr}^{(1)}\) (see Fig. 4). When mode-coupling effects are present, the mode starts to grow at a faster pace if the polarization current effects vanish (black dashed curve). If the effect of mode coupling is weak, small saturated states result (black dot dashed trace in figure 4 for the \(\chi_\perp/\chi_\parallel\) model). For sufficiently strong coupling effects, for both cases, at a given stage the island width may overcome either \(W_{thr}^{(1)}\) or \(W_{thr}^{(2)}\), making irreversible the transition to the large saturated states \((W = 4, \text{ in the a.u. used in Fig. 4})\). Here, irreversibility means that even if the mode coupling interaction vanishes, the driven mode will still continue to grow.

We can argue that a perturbation due to mode coupling will add to \(\Gamma_k^{(0)}(W_k)\) and if sufficiently large will result in a global positive \(\Gamma_k(W_k, W_{k'}, W_{k''}) = \Gamma_k^{(0)}(W_k) + \Gamma_{\text{coupl}}(W_k, W_{k'}, W_{k''}, \cos \Delta \phi) > 0.\) Since as shown in figure 4 a bifurcation process occurs for coupling strength such that the condition \(\Gamma_k(W_k) = 0;\)
\(\partial \Gamma_k(W_k) / \partial W_k = 0\) is met at a small value \(W_k = W_{thr}^{(1)}\).

For a quantitative evaluation it is necessary to consider the (instantaneous) unperturbed non-linear rate of growth of the k-labeled mode:

\[
\Gamma_k^{(0)}(W_k) = \frac{r_s^2}{\tau_{R,k}} \left\{ -|\Delta_0| + \beta_0\left[ \frac{a_b W_k}{W_k^2 + W_d^2} - \frac{a_{polp}}{W_k^3 + W_{p0}^3} \right] + \Re(\Delta_{wall}) \right\}.
\]  

(2)
In the rhs of eq.2 the $|\Delta_{0k}|$ tearing stability parameter in general must be assumed to have the expression appropriate for the helical pitch considered, and the remaining terms describe the conventional neo-classical bootstrap term, the ion polarization current term, and the resistive wall term [9].

The driving mode $(m=1, n=1)$ grows non-linearly to sufficiently large amplitude driven by its own free energy, with an unperturbed rate of the type

$$\Gamma_{1,1}(W) \cong \frac{r_s^2}{0.315 \tau_\rho} \left[ 1 - \frac{A \alpha^2}{s^2} \frac{1}{W + \varepsilon d_e} \right] - C \frac{\eta_0}{\eta_0} \frac{W}{[W + \varepsilon d_e]},$$

$$\alpha = 2 \mu \phi' / B_0^2, \quad d_e = c/\omega_\rho \text{ and } A, C, \varepsilon \text{ are numerical coefficients [15,16]. Then non-linear coupling perturbs the metastable state of the } (m=3, n=2) \text{ mode forcing to positive values the global growth rate, of the type}

$$\Gamma_{3,2}(W_{3,2}, W_{1,1}, W_{4,3}) = \Gamma_{3,2}^{(0)}(W_{3,2}) + \Gamma_{\text{coup}}(W_{3,2}, W_{1,1}, W_{4,3}, \cos \Delta \phi) \quad (3).$$

We assume the three modes satisfy a perfect resonance relation and that $\Delta \phi = 180^\circ$, for maximum interaction [10]. A mismatch in frequency between the natural rotation frequencies of the modes leads to a phase mismatch and reduces the efficiency of the coupling.

The structure of non-linear coupling term of the triplet combination considered is:

$$\Gamma_{\text{coup}}(W_k, W_{k'}, W_{k''}, \cos \Delta \phi) = \frac{r_s^2}{\tau_{R,m}} \frac{r_{s1} C h_k h_{k'} h_{k''}}{W_k W_{k'} W_{k''}} \cos \Delta \phi \quad (4)$$

$$C = \frac{\mu_0}{2} \left\{ \frac{r_{s3}}{r_{s1}} \right\}^{1-m_1} \left\{ \frac{r_{s2}}{r_{s3}} \right\}^{m_2} \frac{R^2 q^2}{m_1 - n_1 q} \left[ \frac{\lambda'}{B_\phi} \right] r_{s3} - \left\{ \frac{r_{s2}}{r_{s1}} \right\}^{1-m_1} \left\{ \frac{r_{s2}}{r_{s3}} \right\}^{m_3} \left\{ \frac{R^2 q^2}{m_1 - n_1 q} \right\} r_{s2} \right\}$$

is the coupling coefficient calculated assuming that each non-linear island is equivalent to a driving current sheet located at rational surfaces $r_{s1}, r_{s2}, r_{s3}$ [10] and that global torque balance is maintained. The other symbols are defined as:
\[ \lambda' \equiv R(J_{0/0}/B_0'), a_p = a_2 \sqrt{\epsilon L_p L_q^{-1}}, a_p(\omega) = a_3(\omega) \left( \rho L_q^{-1/2} \right) g(\epsilon, \nu_i) \text{, } W_d \propto \rho_1^{1/3} \]

\[ W_{th}^2 \propto \rho_i^2 a_2 a_2^{-1/2} L_q L_P^{-1} g(\epsilon, \nu_i) \text{ and } a_2, a_3 \text{ are standard coefficients of the NTM theory [13, 14].} \]

The mode coupling term is increasing in the region of smaller island widths, therefore providing an effective trigger mechanism. If the uncoupled \((m=3, n=2)\) mode were neo-classically stable \((\beta_\theta < \beta_{\theta cr})\), then after the mode \((m=1, n=1)\) disappears \((W_{1,1} \rightarrow 0)\), also the \((m=3, n=2)\) mode would decay. On the contrary if mode \((m=3, n=2)\) when uncoupled meets the condition \(\beta_\theta \geq \beta_{\theta cr}\), and after destabilization due to the three-mode coupling it reaches in a short time interval \(\tau\) a width \(W_{3,2} = \Gamma_{3,2} \tau \geq W_{th}\), then even after the \((m=1, n=1)\) disappears, the \((m=3, n=2)\) keeps growing at a rate \(\Gamma_{3,2} > 0\) and the destabilization is irreversible.

The neo-classical nature of the perturbations must be related to the regime of the discharge. In Table I and II the main parameters of the discharge considered, are reported in the time interval 22.25-22.37 s. In Fig. (3a) the location of \(r_{s1}, r_{s2}, r_{s3}\) is obtained from the EFIT \(q\) profile and the rotation frequency profile (mapped onto the poloidal angle of the magnetic pick-up coil measurement). Fig. 3b shows the measured and the smoothed electron LIDAR pressure profile used in Tables I and II.

**Table I**

<table>
<thead>
<tr>
<th>(I_p)</th>
<th>B</th>
<th>(q_{s5})</th>
<th>(T_e(0))</th>
<th>(n_e(0))</th>
<th>(Z_{eff})</th>
<th>(R_s)</th>
<th>(A)</th>
<th>(\kappa)</th>
<th>(a_2)</th>
<th>(a_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.37 MA</td>
<td>1.37 T</td>
<td>3.64</td>
<td>3.8 keV</td>
<td>2.954 *10^{19} m^{-3}</td>
<td>2</td>
<td>3.12m</td>
<td>0.94 m</td>
<td>1.7</td>
<td>0.816</td>
<td>0.0032</td>
</tr>
</tbody>
</table>

**Table II**

<table>
<thead>
<tr>
<th>((m,n))</th>
<th>(p^*)</th>
<th>(v^*)</th>
<th>(\beta_N)</th>
<th>(L_q)</th>
<th>(L_p)</th>
<th>(\beta_{\psi})</th>
<th>(\beta_{\theta})</th>
<th>(R_s)</th>
<th>(W_{po})</th>
<th>(W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,1</td>
<td>0.35</td>
<td>0.748</td>
<td>2.825</td>
<td>0.666m</td>
<td>1.567m</td>
<td>1.98</td>
<td>3.241 m</td>
<td>.144 m</td>
<td>3.523 m</td>
<td>.021 m</td>
</tr>
<tr>
<td>3,2</td>
<td>0.134</td>
<td>0.535m</td>
<td>0.772m</td>
<td>0.243</td>
<td>3.2</td>
<td>3.523 m</td>
<td>.021 m</td>
<td>3.456m</td>
<td>.033 m</td>
<td>.071 m</td>
</tr>
<tr>
<td>4,3</td>
<td>0.139</td>
<td>0.595m</td>
<td>1.233m</td>
<td>0.656</td>
<td>5</td>
<td>3.456m</td>
<td>.033 m</td>
<td>.071 m</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 2d shows that the modes \((m=1, n=1)\) and \((m=4, n=3)\) are observed at relatively large amplitudes \(B_\theta \sim 2\ \text{Gauss and 0.5 Gauss respectively, at } t=22.25\ \text{s}\) well before the \((m=3, n=2)\) mode is triggered. This corresponds to island sizes \(W_{1,1} \sim 0.15\ \text{m}\) and \(W_{4,3} \sim 0.07\ \text{m}\) estimated from magnetic and \(T_e\) profile perturbations. Within the next 20 ms, the \((m=1, n=1)\) and \((m=4, n=3)\) modes have grown to sufficiently large amplitudes (observed peak amplitudes of \(\sim 5\ \text{Gauss and 2 Gauss}\) respectively) to drive the \((m=3, n=2)\) unstable. The theoretical principle of the mode triggering described by the \(\Gamma \ vs \ W\) plot in Fig. 4 is evaluated in Fig. 5 for the \((m=3, n=2)\) mode with the data of the specific JET example considered (Tables I and II). Curves A-C show the growth rate \(\Gamma_{3,2}(W_{3,2}, W_{1,1}, W_{4,3})\) including coupling as defined in equation (4) above, calculated for three different values of \(W_{1,1}\). The bottom full line represents the unperturbed Rutherford growth rate of the \((m=3, n=2)\) mode, \(\Gamma_{3,2}(0)(W_{3,2})\), that is negative for \(W/a< 0.001\) and is made definitely positive by the coupling perturbation. (The curves shown, are samples from real data, however it should be remembered that the experimental profiles used in Tables I and II are not available at the precise time when the \((m=3, n=2)\) is triggered.)

The same reasoning could be applied in the case of toroidal coupling of the \((m=3, n=2)\) mode with an active \((m=2,n=2)\) harmonic of the \((m=1, n=1)\) having the amplitude one half of the \((m=1, n=1)\). In this case the toroidal coupling term of mode \((m,n)\) with mode \((m+1,n)\) is:

\[
\Gamma_{\text{coupl}}(W_{m+1}, W_m, \cos \Delta \phi) = \frac{r_s^2}{\tau_{R,m+1} R} \left( \frac{r_{sm}}{r_{s(m+1)}} \right)^m \frac{h_m}{h_{m+1}} W_m^2 \cos \Delta \phi
\]

[9] and could produce a comparable destabilization as shown in the Fig. (6). However the condition of frequency matching required in this case, that the driven and driving modes have the same frequency, \(\omega_{(m,n)} = \omega_{(m+1,n)}\), is not satisfied (see Fig. 2a).

Figures 5 and 6 show that for perfect frequency matching conditions, both toroidal and non-linear coupling may destabilize the \((m=3, n=2)\) mode, with driving sawtooth precursor island. There is no need for a seed island since mode coupling may drive the mode unstable, above the threshold for irreversible destabilisation, by forced reconnection.
The occurrence of frequency matching even for a transient time may be sufficient to trigger the bifurcation shown in figure 4. The bifurcation threshold may be below the noise levels, making it difficult to assess experimentally the role of coupling in the destabilisation of the mode. Assessment of a frequency mismatch at the early stages of the forced reconnection requires a more complex calculation than the one provided here. The data shows that the mismatch in the observed frequencies is smaller for non-linear coupling, making it the most plausible mechanism. The non-linear coupling strength increases with mode number [10], thus this mechanism may also explain the onset of \( n > 2 \) modes observed in the JET high performance DT experiments [1]. In the non-linear coupling hypothesis, we assumed for simplicity that the driving mode was the \((m=1, n=1)\), while the smaller amplitude \((m=4, n=3)\) was an idler mode. The data indicates that the \((m=4, n=3)\) mode is far from passive and variations in amplitude and frequency may have played an important role in the NTM onset. It appears that the \((m=4, n=3)\) frequency decrease may have brought the \((m=3, n=3)\) mode marginally stable at \( t \approx 22.35 \) s, while the subsequent \((m=4, n=3)\) growth at \( t \approx 22.36 \) s, together with the large \((m=1, n=1)\) amplitude, may have provided conditions for the NTM irreversible grow.

3. CONCLUSIONS
In conclusion we have given qualitative and quantitative arguments showing that mode coupling can be an effective mechanism to trigger neoclassical tearing modes with mode numbers \( m > 1, n = 1 \) through a process of driven reconnection. A simple theoretical model has been proposed, based on extension of the Rutherford equation, and applied to a particular JET example where the observed frequencies indicate that in this case the relevant coupling is a three mode non-linear one.

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REFERENCES
[3a]. Buttery, R. et al., to be submitted to Nuc. Fus.


Fig.1: Overview of JET pulse 47285, showing the temporal evolution of: a) $D_\alpha$ emission, b) Magnetic signals of the n=1 and n=2 perturbations, c) Central temperature and, d) Neutral beam injection (NBI) power waveform.

Fig.2: a) Magnetic signal measured on the low field side at an angle of 60° above the mid-plane. b) Spectrogram showing the frequency and amplitude of modes observed around a sawtooth crash: the sawtooth precursor (1,1) and its harmonics (2,2) and (3,3), a (4,3) mode unaffected by the sawtooth crash and the onset of a (3,2) mode observed above the noise level at t~22.35s. The (3,2) mode starts to grow irreversibly at t~22.36s.

Fig.2(c): Power Spectra in the time interval 22.344s – 22.349s, immediately after the (m=4, n=3) crash.

Fig.2(d): Temporal evolution of $B_q$ amplitude for the modes seen in figure 2(a), obtained by fft and filter techniques. The spikes seen on the n=1 mode correspond to fishbones, whose frequency is difficult to separate from that of the sawtooth precursor. The n=2 component of the sawtooth precursor shows that the precursor in fact grows continuously up to the crash.
Fig.3(a): Plasma toroidal rotation (charge exchange diagnostic) and q profile (EFIT). The mode frequencies are taken from the spectrogram in figure 2.

Fig.3(b): Measured plasma pressure profile and profile fitting used to calculate the data of Tables I and II.

Fig.4: Illustration of the principle of destabilisation of NTMs driven by mode coupling.

Fig.5: Growth rates for mode \((m=3,n=2)\) without (solid blue) and with non-linear coupling (curves A, B and C) versus normalized island size \(W/a\). The curves A-C where calculated for fixed \(W_{4,3} = 0.071\) cm, for three different \(W_{1,1}\) values \(W_{1,1} = W, W_{1,1} = W/2\) and \(W_{1,1} = W/4\), curves C to A, respectively. The seed island \(W_{rec}/a < 0.001\) is stable and may grow when the driving mode \((1,1)\) reaches \(W_{1,1}/a > 0.035\).
Fig. 6 - Growth rates for mode \((m=3, n=2)\) without (solid blue) and with non-linear coupling (curves A, B and C) versus normalized island size \(W/a\). The curves A-C were calculated for three different \(W_{2,2}\) values (\(W_{2,2} = W_{1} = 0.144\) m, \(W_{1,2} = W_{1}/2\) and \(W_{1,1} = W_{1}/4\), curves C to A, respectively). The seed island \(W_{\text{rec}}/a < 0.001\) is stable and may grow when the \((2,2)\) harmonic of the sawtooth pre-cursor, the driving mode, reaches \(W_{2,2}/a > 0.035\).