A Parametric Model for Fusion Neutron Emissivity Tomography for the KN3 Neutron Camera at JET
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ABSTRACT.
A parametric model for fusion neutron emissivity is presented and applied to the KN3 neutron camera data collected during the Trace Tritium Experiment at the Joint European Torus. This work is aimed at achieving a good compromise between accuracy of tomographic reconstruction and low model complexity. This means low numerical degeneracy and good time-consistency of the results. The model is compared both with plasma simulation codes and other tomographic techniques showing good agreement over the entire data set analyzed.

1. INTRODUCTION
Tomographic reconstruction of two-dimensional neutron emissivity profiles (n m$^{-3}$ s$^{-1}$) is important for retrieving diagnostic information on spatially resolved fusion rates. Experimentally, this requires an array of neutron detectors each placed at the end of a well collimated Line Of Sight (LOS) measuring line integrated neutron rates. At the Joint European Torus (JET) such an instrument is the KN3 neutron camera [1], featuring 10 horizontal and 9 vertical LOS covering a section of the tokamak. Two neutron detectors are installed in each KN3 LOS, namely a NE-213/BC-501 [2] liquid and a BC-404 [3] plastic scintillation detector. These are used for detection of neutrons from deuterium-deuterium (DD, $E_n \approx 2.45$ MeV) and deuterium-tritium (DT, $E_n \approx 14$ MeV) fusion reactions, respectively. This paper focuses on BC-404 DT data collected during the Trace Tritium Experiment (TTE) in 2003 and in particular for the JET Pulse No’s: 61114-61633. This range was chosen since it represents the majority of the TTE data and because there were no changes in the KN3 detector calibration, which can potentially affect the results.

2. EXPERIMENTAL DATA
The KN3 raw data is available at fixed 10 ms intervals and provides the number of counts for each of the 19 KN3 channels ($c_i$, $i = 1; 2; ..., 19$). The sum of the counts of all channels in the same time interval is indicated with $c_{\text{tot}}$. In this work contiguous time intervals were dynamically merged to achieve $c_{\text{tot}} \approx 3000$ counts and these aggregations are here referred to as time slices. The value chosen for $c_{\text{tot}}$ was found to be a good compromise between low statistical fluctuations and good time resolution. A response function $R_i$ was calculated for each channel to map a given emissivity profile $E$ into neutron rate at each detector (n/s). Detector counts were estimated by multiplying 41 this by the time slice duration ($\Delta t$) and by the corresponding detector efficiency ($\eta_i$) obtained through the KN3 calibration. The process of mapping $E$ into detector counts for all channels is here referred to as folding, as in equation (1). Both $R_i$ and $E$ are usually defined on a discrete grid $G$ which explains the use of a sum in place of an integral in equation (1). The index $p$ in this case refers to any point of $G$.

$$c_d \equiv \eta_i \Delta t \sum_{p \in G} E_p R_{i,p} \quad i = 1, 2, ..., 19$$ (1)
was calculated geometrically by mapping the solid angle of each detector from given points in the torus. A preliminary comparison with Monte Carlo simulations (MCNP [4]) suggested that this approximation was adequate, at least for the direct neutron fluxes. \( R_i \) however did not account for any scattered fluxes such as those generated by neutrons scattered in the collimator (in-scatter) or reaching the detector after scattering from the JET wall (back-scatter). Modeling these effects is necessary for a more accurate description of the KN3 data and typically requires full-scale Monte Carlo simulations. A detailed MCNP model for KN3 and the relevant JET sector is being developed for generating a more accurate response function which can account for both direct and scattered fluxes.

2.1. FRAMES OF REFERENCE AND GRIDS

Two frames of reference were used to describe the data. The first one \((\Omega_{\rho\theta})\) is polar and assumes circular magnetic flux surfaces of radius \(\rho\) and angle \(\theta\). \(\rho\) varies from 0 (magnetic axis) to 1 (Last Closed Flux Surface, LCFS) while \(\theta\) extends over the region \([-\pi, \pi]\). The second frame of reference \((\Omega_{RZ})\) is cartesian and uses the physical JET coordinates for major radius (R) and height (Z). The magnetic axis in \(\Omega_{RZ}\) is indicated as \((R_{\text{MAG}}, Z_{\text{MAG}})\).

The relationship between \(\Omega_{RZ}\) and \(\Omega_{\rho\theta}\) depends on the magnetic configuration of each time slice and it is numerically provided (point-wise) at JET by the software codes FLUSH [5] and EFIT [6]. This means that a grid \((G_{RZ})\) in \(\Omega_{RZ}\) has a corresponding grid \((G_{\rho\theta})\) in \(\Omega_{\rho\theta}\). For the present study \(G_{RZ}\) ranged from \(Z = -1.99\text{m}\) to \(Z = 2\text{m}\) and \(R = 1.80\text{m}\) to \(R = 4\text{m}\) at uniformly spaced 1cm intervals. This yielded a matrix with 400 rows and 221 columns. \( R_i \) was calculated for the same grid for all channels. The mapping between \(G_{RZ}\) and \(G_{\rho\theta}\) was implemented on a matrix index-by-index basis for both grids. This and other arrangements in section 3 were chosen for fast execution time in the code MATLAB [7] which was adopted for this application.

3. A PARAMETRIC MODEL FOR NEUTRON EMISSIVITY

3.1. THERMAL

The Thermal component describes the thermal neutron emission from the plasma. This means ion kinetic temperatures typically between 2keV and 30keV with a nearly uniform distribution of pitch angle, i.e. the angle between a particle velocity and the magnetic field [8]. Simulations showed that under these conditions the emissivity is usually almost constant along the magnetic flux surfaces and can be parametrized exclusively in \(\Omega_{\rho\theta}\) as a function of \(\rho\). Two contributions are considered, namely a Gaussian center with standard deviation \(\sigma\) and a quasi-parabolic edge [9] as in equation (2). An additional parameter \(\text{ATHERM}\) is used to set the overall amplitude. The parameter is referred to as peaking factor and \(f\) describes the ratio between the Gaussian and the quasi-parabolic contributions \((0 \leq f \leq 1)\). The addition of a Gaussian center allows the model to approximate profiles with rapid changes in rst derivative such as those associated with Internal Transport Barriers (ITBs) and H mode plasmas. This functional form is not unique but appeared to be a good compromise between simplicity and accuracy in describing the data.
Equation 2 can be applied to all points of $G_{\rho \theta}$ in a single step using matrix operators and then be displayed on $G_{RZ}$. This allows the emissivity profile to inherit the proper ellipticity, triangularity and Shafranov shift profiles [8].

3.2. NEUTRAL BEAMS AND TRAPPED PARTICLES

The Neutral Beams (NB) component describes the neutron emissivity from non-thermalized particles in the plasma generated by Neutral Beam Injection (NBI) heating. The modeling is based on three main assumptions. The first ($H_1$) is that the toroidal magnetic field is dominant and the poloidal field can be neglected for the calculation of the turning point of trapped particles as well as for the estimation of the trapped/passing boundary. This is in general a good approximation for tokamaks [10]. The second assumption ($H_2$) is that the NBI deposition is mostly located on the magnetic mid-plane $Z = Z_{MAG}$ and all ions have the same effective pitch angle $\phi_{eff}$. The third ($H_3$) is that trapped particle orbits are narrow and can be collapsed to a line on a flux surface. While $H_2$ and $H_3$ are not realistic for the motion of a single ion [11], numerical simulations suggested that they become meaningful for a large ensemble of particles. This aspect is discussed further in section 3.2.5. The generation of the NB component is rather complex and is described through four steps in what follows, using both the $U_{\rho \theta}$ and $U_{RZ}$ frames of reference.

3.2.1. Step 1

In compliance with $H_1$ the major radius of turning point $R_{tp}$ for a particle deposited at a point $(R;Z)$ with pitch angle $\phi$ can be estimated by conserving the magnetic momentum [11] as in equation (3).

$$R_{tp} \sim R \sin^2 (\phi)$$

The guiding center trajectory [11] for a trapped particle is qualitatively illustrated in figure 1 (red solid line). The flux surfaces are displayed with dotted concentric lines sampled at regular intervals in $\rho$. According to $H_3$ the trajectory can be collapsed to a line lying on a flux surface such as the bold black line. This simplification is not strictly necessary, but provides significant computational advantages. Consider the point P in figure 1, which is defined as $(R_p, Z_p)$ in $U_{RZ}$ and $(\rho_p, \theta_p)$ in $U_{\rho \theta}$. From $H_2$ and $H_3$ it follows that P was deposited in $A \equiv (R_A; Z_{MAG}) \equiv (\theta_p, \theta_T)$ with pitch angle $\phi_{eff}$ and is thus associated with the turning point $T \equiv (R_T, Z_T) \equiv (\theta_p, \theta_T)$. The latter can be calculated through equation (3). The linear ($\delta_r$) and angular ($\delta_\theta$) distance from P to T (figure 1) can be determined by means of simple matrix operations. These parameters are necessary for the next steps of the NB component. The same approach can be applied to any point on the bold line as well as to any point in the grid in a single step. Without $H_3$ it would be necessary to integrate the differential equations.
of motion for each particle which requires orders of magnitude longer execution time.

3.2.2. Step 2
Repeating the procedure described in section 3.2.1 for an NBI deposition profile $\Gamma_\rho$, rather than for a single trajectory, produces an emissivity profile such as figure 2a. Here $\Gamma_\rho$ was modeled using a Gaussian peaked at $\rho = \rho_{peak}$ with asymmetric standard deviation $\sigma$ as in equations (4) and (5). $\sigma_{in}$ is the inner ($\Gamma_\rho$ peak) standard deviation while it is assumed that the outer ($\rho > \rho_{peak}$) standard deviation is proportional to the distance between $\rho_{peak}$ and 1, i.e. the LCFS. $K$ is a numerical constant usually set to $\approx 2 - 3$ to simplify the model and to remove one degree of freedom. This simple functional shape was derived from simulations using the actual JET NBI geometry and ionization cross sections [12].

The deposition profile is drawn in figure 2 with the bold black line peaking around $R \approx 3.6 - 3.7m$ ($\rho_{peak} = 0.7$, $\sigma_{in} = 0.4$, $K = 3$, $\varphi_{eff} = \pi/3$). It is assumed that the amplitude of the emissivity profile along $\rho$ is given only by $\Gamma_\rho$.

$$\Gamma_\rho = e^{- (\rho - \rho_{peak})^2 / 2\sigma^2}$$  \hspace{1cm} (4)

$$\sigma = \begin{cases} \sigma_{in} & \rho = \rho_{peak} \\ (1 - \rho_{peak})/K & \rho > \rho_{peak} \end{cases}$$  \hspace{1cm} (5)

3.2.3. Step 3
The abrupt termination of the trajectory at the turning point (step function) such as in figure 2a is not physical for a large ensemble of particles. A sigmoid $\Sigma_\delta$ has therefore been used in place of the step function across the turning point. The distance to the turning point $\delta_R$ (taken along the R direction) has been used as the argument of the sigmoid as in equation (6). Figure 2b shows the profile of figure 2a after the multiplication by $\Sigma_\delta$. $\tau$ was set to 0.1 for this example.

$$\Sigma_\delta = \frac{1}{e^{-\delta_R/\tau} + 1}$$  \hspace{1cm} (6)

3.2.4. Step 4
This step accounts for the fact that trapped ions spend more time near the turning point, where the component of the velocity parallel to the magnetic field (v||) tends to zero. Here the emissivity is expected to be a factor $G_\theta$ higher than on the magnetic mid-plane. This factor is here referred to as the poloidal gain. $G_\theta$ can be estimated analytically for a single particle from the inverse of $v_||$. For an ensemble of particles, however, this is no longer possible due to the statistical broadening of parameters such as position, energy and pitch angle. One possible parametrization for $G_\theta$, suggested by simulations, is illustrated in equation (7) using powers ($\alpha$) of sine of argument $\propto \theta/|\delta|$ and a
variable $\Lambda_G$ to set the magnitude of the gain. The additional factor between $\Lambda_G$ and the sine is a sigmoid of argument $(\rho - \rho_{\text{crit}})$ and parameter $\lambda$ used to smooth $G_\theta$ around the trapped/passing boundary $\rho_{\text{crit}}$. The latter can be estimated from the conservation of the magnetic momentum using the assumption $H_1$ and $H_2$ [10]. The parameters $\alpha$ and $\lambda$ are usually set to $\approx 2$ and $\approx 0.05$ to reduce the number of free variables in the model.

$$G_\theta = 1 + \Lambda_G \left( 1 + e^{-(\rho - \rho_{\text{crit}})/\lambda} \right) \left( \sin \frac{d\theta}{2|\delta_\theta|} \right).$$  \hfill (7)

Figure 2c shows the profile of figure 2b after the multiplication by $G_\theta$, using $\Lambda_G = 1$. This is the completed NB component and features 5 degrees of freedom ($\varphi_{\text{eff}}, \rho_{\text{peak}}, \sigma_{\text{in}}, \tau$ and $\Lambda_G$) as well as three parameters which are usually kept fixed ($K, \alpha$ and $\lambda$). An additional variable ANB is used to set the overall amplitude of the $A_{\text{NB}}$ component, as in equation (8).

$$s = A_{\text{NB}} \Gamma_\rho \Sigma_\delta G_\theta$$ \hfill (8)

3.2.5. Comparison with simulations

The approximations $H_1, H_2$ and $H_3$ allowed for performing the necessary steps for the NB component through simple matrix operations and high computational efficiency. Nevertheless, the validity of $H_2$ and $H_3$ for a large ensemble of particles was tested by calculating the guiding center motion trajectories for a large number of particles (several thousands) using realistic NBI deposition profiles. This made use of first-order approximations for the drift velocity [10] and of a parallelized Runge-Kutta 4th order (RK4) scheme [13] for numerical integration. One of the simulation results is shown in figure 3 in the left plot while a fit to the same emissivity profile using the NB model is illustrated in the right one. The low ellipticity and triangularity of the magnetic configuration were chosen to match the approximations adopted in the expression of the drift velocity and in the integration. The similarity of the fit and the RK4 calculation can be considered an indication that $H_2$ and $H_3$ are valid approximations for a large ensemble of particles and orbits. Results obtained with other simulations, JET TTE experimental data and through some comparisons with TRANSP [14][15] using the NBI module NUBEAM [16] seem to confirm this statement.

3.3. GENERALIZED TRAPPED/PASSING COMPONENT

The Generalized Trapped/Passing (GTP) component describes both trapped and passing particles and is to some extent a generalization of the NB component. It is based on an asymmetric Gaussian ring of radius $\rho_{\text{peak}}$ which shares the same functional shape and notation as equation (4). $K$ is set to $\sim 2-3$ as discussed in section 3.2.2. The center of the ring can be shifted through a parameter $\epsilon R$ to account for the drifts acting on the passing particles orbits [11]. From simulations $\epsilon R$ is expected in most cases to be relatively small ($\sim$ few cm) for passing particles near the magnetic
axis. Similarly to the NB component, the GTP component features a poloidal gain $G_{\theta}$. A possible expression for this uses two Gaussians centered at $\theta = +\nu$ and $\theta = -\nu$ with standard deviation $\sigma_{\nu}$. These can be combined in two curves $\gamma_1$ and $\gamma_2$ as in equations (9) and (10). Summing $\gamma_1$ and $\gamma_2$ and subtracting the minimum value of the sum yields a smooth curve with periodic boundary conditions such as in equation (11).

$$\gamma_1 = \exp\left(-\frac{(\theta - \nu)^2}{2\sigma_{\nu}^2}\right) + \exp\left(-\frac{+2\pi - \nu}{2\sigma_{\nu}^2}\right)$$

$$\gamma_2 = \exp\left(-\frac{(\theta + \nu)^2}{2\sigma_{\nu}^2}\right) + \exp\left(-\frac{+2\pi + \nu}{2\sigma_{\nu}^2}\right)$$

$$\gamma = \gamma_1 + \gamma_2 - \min (\gamma_1 + \gamma_2)$$

$G_{\theta}$ was generated by normalizing $\gamma$ to its maximum value with the addition of a pedestal factor $P$ as in equation (12)

$$G_{\theta} = \gamma / \max (\gamma) + P$$

Finally, the GTP component is generated by multiplying $\Gamma_{\rho}$ by $G_{\theta}$, normalizing and applying an amplitude factor $A_{GTP}$, as in equation (13).

$$s = A_{GTP} \Gamma_{\rho} G_{\theta}$$

This component features 6 degrees of freedom besides the amplitude, namely $\rho_{\text{peak}}$, $\sigma_{\text{int}}$, $\nu$, $\sigma_{\nu}$ and $P$. In some cases of dominant passing particle contributions, such as o-axis high field side (HFS) beam deposition by tangential injectors (e.g. JET Pulse No's: 204 61235 or 61237), the parameter $\nu$ can be fixed at $\pi$ since in such a case the profile would show a single lobe towards the inner side of the torus. $\epsilon_R$ can also be estimated from calculations and then kept constant for a further reduction of the degrees of freedom.

### 3.4. ICRH

This component provides a simple model for the Ion Cyclotron Resonance Heating (ICRH) interacting with the plasma along a resonance layer $\Psi$. This is modeled by a Gaussian gain factor $G_{\Psi}$ defined on the tokamak major radius $R$ with standard deviation $\sigma$. An additional parameter $K$ allows for asymmetric behavior of the gain factor with respect to $\Psi$. Fixing $K$ to unity generally yields good results and removes one degree of freedom.
The ICRH contribution is obtained by the multiplication between $G_R$ and a base component. This is usually the Thermal component as defined in section 3.1. Application of $G_R$ to a more complex model such as NB or GTP usually leads to numerically unstable results at low counts due to numerical degeneracy. This component was found to represent the experimental data relatively well despite its simplicity. This was at least the case for the DT BC-404 data for KN3 during the TTE campaign.

### 4. RESULTS

#### 4.1. ANALYSIS

Performance analysis was based on quality of indicators such as Cash Statistic ($C_{stat}$) \[18\] and reduced chi square ($\chi^2$) between the data and the folded emissivity profile ($Y$) The volume integral of the neutron emissivity was normalized (norm$_1$) to the total DT neutron rate estimated from silicon diode data\(^\dagger\). $Y$ was area-normalized (norm$_2$) to the number of counts in the time slice. The reason for both normalizations was to focus exclusively on the shape of the profile, rather than on the absolute values, allowing for a proper assessment of the validity of the functional forms proposed by the model. This also allowed for a more consistent application of $C_{stat}$ and $\chi^2$. Tests performed without norm$_1$ and norm$_2$ produced results which were often within a relatively small multiplicative factor from the data, e.g. \(\sim 10-30\%\). Nevertheless, this would cause $Y$ to lie either systematically above or below the data producing high values of $C_{stat}$ and $\chi^2$ despite an adequate shape of the $t$. Once the MCNP response function discussed in section 2 is available the model would likely be more applicable also at an absolute level, i.e. without the normalizations norm$_1$ and norm$_2$.

The use of $C_{stat}$ represented the most conservative approach since it assumed that the Poisson fluctuations in the counts of each detector channel were the only source of errors. All errors concerning the magnetics (flux surface shapes and magnetic axis), detector calibration, response function and so forth were assumed to be null. The adoption of $\chi^2$ provided a simple way to account for these additional sources of errors by summing their contributions in quadrature. In this paper only two error sources were considered for $\chi^2$. One was the statistical uncertainty assuming a Gaussian distribution of counts (rather than Poisson), while the second represented the nominal KN3 calibration errors $\epsilon_i \ (\approx 3-10\%)$ \[19\] as in equation (15) with $c_i$ being the raw 245 counts in the $i$-th channel and $\sigma_i$ the respective error bar.

$$\sigma_i^2 = (\sqrt{c_i})^2 + (\epsilon_i c_i)^2$$  \[(15)\]

The results provided in this section were achieved by iteratively varying the model parameters until the minimum $\chi^2$ was reached. Minimization of $C_{stat}$ yielded similar results. No weights were applied in the calculation of either metric and all channels were assumed to share equal relevance for the fit.

\(^\dagger\) JET pulse processed data (PPF) node M7TT/R14
4.2. PERFORMANCE OF THE MODELS

Figure 4 (left) provides an example for the Thermal+NB model during a brief tritium neutral beam injection (T blip) onto a high density deuterium plasma (JET Pulse No’s: 61433, 20.080 20.100 s). The TRANSP simulation for a similar time slice, i.e 20.075 20.100s, is reported in the center with good agreement on a relative scale. The 5ms difference at the beginning of the time slice is due to the different time steps between the TRANSP NUBEAM module (25ms) and the KN3 raw data (10ms). Both emissivity profiles have been folded by the KN3 response function, area normalized to the number of counts in the data (norm2) and compared to the raw data in figure 4 (right). This produced $\chi^2$ values of 1.05 and 1.87 for the model and TRANSP respectively. Cstat values were slightly higher at 2.23 and 3.69 for the reasons explained in 4.1.

Figure 5 (left) reports another example of application of the model for high field side (HFS) deposition of a T blip (JET Pulse No’s: 61237, 6.220–6.270s) using the GTP component only. The result is compared to an established tomographic code, TOMO5 [20] (center), showing good qualitative agreement. When compared to the data (right), the folded profiles show $\chi^2$ values of 1.41 and 4.03 for the model and TOMO5 respectively (2.25 and 4.42 for Cstat). TRANSP simulations are also available for this JET pulse [21] and qualitatively confirm the results displayed above.

An example of the application of the ICRH model is given in figure 6 on JET Pulse No: 61349 (5.470–5.550s). This time slice features an on-axis T blip in a plasma with a strong ITB whose outer border is at $R \approx 3.2m$. ICRH was applied with the 3rd harmonic resonance layer for T set at $R \approx 3.1m$. The figure suggests a possible interaction between the fast T beam ions and 3rd harmonic ICRH near the ITB boundary and the T resonance layer, showing a band of stronger neutron emission around this position. The folded solution is compared to the data in the bottom panel of figure 6 showing $\chi^2$ and Cstat values of 1.78 and 2.20 respectively. Comparison with TOMO5 showed a qualitative agreement with this tomogram [22].

Results from the application of the model to the entire pulse range 61134-61633 279 ($\approx 10,000$ time slices) are shown in figure 7 both in terms of $\chi^2$ (solid line) and 280 Cstat (dashed line). Multiple combinations of components were applied to all the data and the one achieving best $\chi^2$ was automatically selected and used to build the statistical distributions in figure 7. Statistical details of the $\chi^2$ and Cstat distributions are presented in Table 1 including the quantiles at 95% (q95%) and 99% (q99%). The most probable $\chi^2$ and Cstat values were found close to 1 (1.27) with a slightly higher mean value for both, i.e. 1.57 and 1.76 respectively. Standard deviation was near unity in both cases. Results showed that 95% of the emissivity profiles could be reconstructed with $\chi^2$ and Cstat below 2.94 and 3.62 respectively, while 99% of the profiles featured $\chi^2 \leq 5.51$ Cstat $\leq 5.81$. As expected, the Cstat values are slightly higher than $\chi^2$.

5. DISCUSSION

The systematic application of the model on a large number of time slices indicated that achieving lower $\chi^2$ or Cstat does not necessarily mean better overall performance. In fact, while fit performance
generally improves with the number of degrees of freedom in the model, the results usually cease to be time-consistent for very complex models. This means large fluctuations in the model parameters which are not consistent with physical considerations or structures appearing and disappearing in the emissivity profile without a physical reason. The model described in this paper was found to be a good compromise between accuracy, time-consistency and low degeneracy. The possibility to fix some parameters as suggested in section 3 permits to reduce degeneracy even further at a relatively small price in terms of $\chi^2$ (an increase of about 0.3-0.5) [23].

The use of $\chi^2$ as a quality of fit parameter provided a simple method to account for errors that are not simple statistical fluctuations in the data. Nonetheless, the use of a quadrature sum for calibration errors as in equation (15) is not an optimal choice since these typically represent a class of systematic rather than statistical errors. In the case of KN3, however, factors like gain drifts over time, event rate and temperature can add stochasticity to the calibration error thus partly justifying the choice adopted. The application of Cstat is more statistically rigorous but by neglecting any other sources of errors than the Poisson fluctuations in the data the results typically appear worse than they should.

Another important class of errors is represented by the uncertainties in the magnetic configuration, since an error in for instance $R_{\text{MAG}}$ or $Z_{\text{MAG}}$ would misplace the toroidal axis of the emissivity profile. This could give rise to relatively large errors in the data, in particular for rapidly varying emissivity profiles in regions of sharp response function gradients. It was not straightforward to quantify these contributions since they are both profile and position dependent. For this reason they were not included in the estimation of the error bars $\sigma_i$ in equation (15). The adoption of such errors would increase the value of $\sigma_i$ and thus reduce $\chi^2$.

The possibility to represent neutron emissivity through a parametric model provides some advantages over pixel-wise codes such as TOMO5. The application of the model produces results, i.e. the model variables, which can be related to physical quantities and which are time-resolved without any further analysis of the tomogram shape. TOMO5, however, might be a better choice to detect structures in the data not yet contemplated by the model. The two techniques could therefore be applied side-by-side for offline analysis. For real-time applications the use of a model-based tomographic reconstruction is likely a more efficient choice, especially when used in combination with artificial neural networks [23]. In this case the tomograms could be reconstructed typically orders of magnitude faster than with pixel-wise codes.

Finally, the parametric model described in this paper could in principle be extended to the DD case without major modifications. A systematic study of such an application will be performed as the aforementioned MCNP response function for KN3 becomes available.

CONCLUSION
This paper has presented a parametric model that can adequately represent the neutron emissivity space through the use of a few parameters. The model was found to be a good compromise between accuracy and low numerical degeneracy. The adoption of $\chi^2$ as a performance parameter allowed
for a simple and preliminary inclusion of some of the experimental errors, although this was always used in combination with the more rigorous Cash Statistic. The systematic application of the model on a wide range of pulses and time slices proved good and consistent performance of the model over a broad spectrum of plasma configurations, heating and fueling scenarios. The model used for neutral beams interaction with the plasma and for trapped particles was found to be rather successful and provided good agreement with simulation codes such as TRANSP. The ICRH model was also found to yield good performance despite its simplicity. Finally, the tomographic results were found to be consistent with established reconstruction codes such as TOMO5. The advantage of a parametric approach in this case lies in producing time-resolved results that are related to physical parameters in the plasma without requiring any further interpretation of the emissivity profile.

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**REFERENCES**

Table 1: Statistical analysis for the $\chi^2$ and Cstat distributions obtained by applying the model to all TTE JET pulses between 61134 and 61633 (=10,000 time slices). Also reported are the 95% and 99% quantiles ($q_{95\%}$ and $q_{99\%}$) of the distributions.

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Figure 1: A section of JET showing the magnetic flux surfaces at uniform intervals in $\rho$ (dotted) and a guiding center trajectory of a trapped ion (solid red). The bold black line represents the collapsed trajectory on a flux surface (hypothesis H3). P is a point of the trajectory, A is the deposition point (hypothesis H2) and T is the turning point. $\delta_R$ and $\delta_\theta$ are the linear (along $R$) and angular distances between P and T.
Figure 2: (color online) Emissivity contour plots of the NB component at steps 2, 3 and 4 (panels a, b and c respectively). The color scale varies linearly from the minimum (dark blue) to the maximum (dark red). The bold black line represents the shape of the NBI deposition profile $\Gamma_\rho$ assumed for this example.

Figure 3: (left) Example of an emissivity profile calculated by RK4 integration of particle orbits from a realistic JET NBI deposition profile. (right) Fit of the emissivity profile in the left plot through the NB model. In both cases the contours are equally spaced and range from dark blue (minimum) to dark red (maximum).
Figure 4: High-density deuterium plasma during a T blip (JET Pulse No: 61433, 20.080–20.100s). (left) Tomographic reconstruction of the emissivity profile through the Thermal+NB model. (center) TRANSP results for a similar time slice, i.e 20.075–20.100s. (right) Folded emissivity profiles from the model and TRANSP. Values of Cstat and $\chi^2$ are displayed in the legend.

Figure 5: Off-axis T blip with HFS deposition (JET Pulse No: 61237, 6.220–6.270s). (left) Tomographic reconstruction of the emissivity profile through the GTP model. (center) TOMOS results for the same data. (right) Folded emissivity profiles from the model and TOMOS. Values of Cstat and $\chi^2$ are displayed in the legend.
Figure 6: On-axis $T$ blip during ICRH and in presence of an ITB (JET Pulse No: 61349, 5.470 - 5.550s). (above) Tomographic reconstruction of the emissivity profile through the ICRH model applied to the Thermal component. (below) Folded solution compared to the experimental KN3 data. Values of Cstat and $\chi^2$ are displayed in the legend.

Figure 7: $\chi^2$ performance for the model applied to all TTE JET pulses between 61134 and 61633 ($\approx$10,000 time slices).