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The Dependence of the Damping Rate of Medium-n Toroidal Alfvén Eigenmodes on the Edge Plasma Elongation in JET

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\textsuperscript{*} See annex of F. Romanelli et al, “Overview of JET Results”, (Proc. 22\textsuperscript{nd} IAEA Fusion Energy Conference, Geneva, Switzerland (2008)).

Preprint of Paper to be submitted for publication in Nuclear Fusion Special Issue
ABSTRACT.
A new set of compact in-vessel antennas has been built and installed in JET to provide for the first time the direct measurement of the damping rate ($\gamma/\omega$) of stable Alfvén Eigenmodes (AEs) with toroidal mode number ($n$) in the range $|n| = 3 \div 15$. This paper reports the first quantitative analysis of the measurements of the damping rate for these modes as function of the edge plasma elongation ($\kappa_{95}$). We find that the scaling of $\gamma/\omega$ versus $\kappa_{95}$ for medium-$n$ Toroidal AEs, with $n = 3$ and $n = 7$, follows the same trend previously measured and explained theoretically for the $n = 1$ and $n = 2$ modes. Theoretical analysis of these measurements has been performed using the LEMan code, with the results in good agreement for the magnetic configurations where there is only a very minor up down asymmetry in the poloidal cross-section of plasma. These experimental results further confirm the possibility of using the edge shape parameters as a real-time actuator for control of the stability of alpha-particles driven AEs in burning plasma experiments, such as ITER.

1. INTRODUCTION
One of the most important physics issues on the way to a fusion reactor is the understanding and the control of burning plasmas, the operational regime where the energy carried by the fusion produced alpha particles ($\alpha$s) exceeds that which has been externally injected to initiate the thermonuclear fusion process. Burning plasmas are characterized by a very strong coupling between their various operational elements, such as the pressure profile of the background plasma, which drives the fusion reactivity but may also cause the onset of magnetic instabilities, and the distribution in phase space of the fusion-born $\alpha$s and their interaction with the background coherent and turbulent instability spectrum. The present-day fusion experiments, such as JET, approach this problem by investigating separately the individual elements of this regime, namely increasing the fusion gain and controlling the background current and pressure driven magnetic instabilities, before actual burning plasma conditions can be achieved in ITER.

One of these elements is the resonant interaction of the $\alpha$s with coherent plasma waves that can be produced by the $\alpha$s themselves due to excessive peaking of their pressure gradient. This interaction can lead to an efficient energy and momentum exchange between the waves and the $\alpha$s [1, 2]. If this mechanism leads to a significant spatial re-distribution of the $\alpha$s themselves up to vessel walls, not only the overall fusion performance will be limited, but also the machine integrity may be affected. Between the various modes that can resonantly interact with the $\alpha$s, Alfvén Eigenmodes (AEs) occupy a special place as they sit in a relatively quiet portion of the plasma electromagnetic fluctuation spectrum, which is well above the frequencies related to drift instabilities driven by a gradient in the background plasma pressure and current, and also well below the ion and electron cyclotron frequency. Hence, AEs represent a direct and clean method of “communication” with the $\alpha$s, from which not only information on their properties can be obtained, but also on those of the core plasma, as it is these properties that have produced the fusion burn leaving a fingerprint on the phase-space distribution of the $\alpha$s themselves.
Between all the different classes of AEs [3], of particular interest are AEs with toroidal mode number (n) in the range n~3÷20, as these are expected to interact most strongly with the αs [4]. The stability of AEs with these medium-n mode numbers is investigated experimentally in JET using a new set of compact in-vessel antennas [5] which provides a direct measurement of the damping rate (ω/γ) as function of the background plasma parameters for individual toroidal mode numbers. These measurements are now being routinely obtained in JET in different operating scenarios. In this paper we report the first quantitative analysis of the dependence of the damping rate on the edge plasma elongation (κ₉₅), which has prompted some direct comparison with code predictions.

This paper is an extended version of a contribution presented at the 11th IAEA TCM on energetic particles [6]. A companion paper [7, 8] reports on some of the technical aspects of the system and on other measurements of ω/γ, and the Readers are invited to refer to this work for further details. This presentation is organised as follows. Section2 gives a brief description of the new JET AE active antenna system. Section3 reports the first quantitative measurements of the damping rate for Toroidal AEs (TAEs) with n = 3 and n = 7, focussing particularly on the dependence vs. the edge elongation. In Section4 we present the first simulations that have been run to model these data using the LEMan code [9, 10]. Finally, in Section5 we present the conclusion of this work and an outline towards future activities.

2. THE NEW ACTIVE ANTENNA SYSTEM FOR EXCITATION OF MEDIUM-n ALFVÉN EIGENMODES

The measurement of the damping rate of AEs has been a long-standing feature of the JET experimental program: the first data were indeed collected for low-n TAEs (n = 0, |n| = 1, |n| = 2) since the mid-nineties using the saddle coil system [11]. This system had one main limitation, namely the possibility of driving only modes up to |n| = 2 because of the in-vessel geometry of the saddle coils. After many years of successful operation, during which we obtained in excess of 10⁵ individual damping rate measurements and performed many different experimental scans, the JET saddle coil system was dismantled during the 2004-2005 shutdown. A new antenna system has been built and installed to replace it. The new antenna system is designed to overcome the n-number limitations of the saddle coils. It is capable of exciting magneto-hydrodynamics modes in the Alfvén frequency range (10→500kHz), keeping similar operational capabilities (I_ANT~15A, V_ANT~1kV, maximum power ~5kW) but now focussing specifically on the medium to high range of toroidal mode numbers |n| = 3÷20 [5]. The system comprises two assemblies of four toroidally closely-spaced coils each, situated at opposite toroidal locations. Figure1 shows a photo of one of these two assemblies as installed in JET.

The maximum antenna current and voltage used for the typical AE excitation experiments are around 10A and 500V, producing a very small antenna-driven radial magnetic field at the position of the antennas, |Bᵣ| ~ 0.5G per 1A of antenna current, in turn driving a typical |Bᵣ| ~ ~0.1G at the plasma edge and |Bᵣ| ~ 0.01G in the plasma core at maximum antenna current. The plasma response
to the driven perturbation is measured using synchronous detection. A 1kHz-clock digital control system, the Alfvén Eigenmode Local Manager (AELM), is used to control the AE excitation in real time. The AELM sweeps linearly the antenna frequency around an initial guess for the AE resonance. The AELM is also used to detect and track in real-time the individual resonances corresponding to antenna-driven, stable plasma modes. A fit of the complex antenna/plasma transfer function is then applied, to obtain the mode frequency and damping rate, as well as the mode amplitude at the different probe locations, with a typical time resolution of the order of 10-20ms. Any combination of the 8 antennas can be chosen with different relative phasing (+ and −) to excite different n-spectra, with measurable radial Br field up to |n| \( \sim 20 \), as shown in fig.2(a-d). Note that a very broad antenna spectrum is excited, comprising many toroidal components, of which the higher-n ones are more strongly attenuated as function of the distance from the antennas. This has been the main motivation towards developing the use of a real-time and post-pulse algorithm for mode detection and discrimination using the Sparse Signal Representation method and the SparSpec code [12], as other algorithms, for instance those based on the Single Value Decomposition (SVD) method, have been found to be inadequate to meet both the real-time and post-pulse measurement requirements. Finally, as shown in fig3, code calculations performed with LION [13] code also demonstrate that this arrangement provides, for the same JET equilibrium, a coupling to the plasma for an n = 5 TAE similar to that achieved with the much bigger saddle coils for an n = 2 TAE.

The damping rate of |n| \( \leq 2 \) modes in ohmic limiter plasmas was found to be essentially identical to that measured with the previous saddle coil system [14, 15], confirming the robustness of the measurements made with the new antenna system. Despite the much lower radial magnetic field driven by the antennas on the magnetic axis (at [R, Z] = [3, −0.3]m we have |B_r| \( \sim 1 \times 10^{-3} \)G for n = 5 compared to |B_r| \( \sim 1 \times 10^{-2} \)G for n = 1 and n = 2), a new result was immediately observed, namely that many marginally stable harmonics with |n| \( \sim 3-12 \) and \( \omega/\gamma < 0.2\% \) were found to be simultaneously excited at very close-by frequencies in the plasma, i.e. a frequency-degenerated mode spectrum. This has prompted the development of a more sophisticated algorithm for mode-number recognition based on the sparse signal representation, based using the SparSpec code [12]. This algorithm has been adapted from its original real-valued data for astronomy and astrophysics applications to deal with complex data in fusion plasmas, and has been successfully validated and benchmarked on real and simulated JET data [16]. A clear illustrative example of the findings on this degenerate mode spectrum and on the data analysis method used to discriminate between its various components is shown in fig.4(a,b).

While in the spectrogram (fig.4a) we can only identify three distinct bands where modes are actively driven and damping rate can be measured, indicated by the much smaller width of the sweep of the (triangular) antenna frequency waveform, it is only with the SparSpec code that we can ascertain that three separate n = 5, n = 7 and n = 11 modes co-exist in the lowest frequency band around 160kHz (fig.4b). Considering this frequency band, had we analysed it using a standard linear phase fitting algorithm to determine the mode numbers, we would have found an n=8 mode.
with a damping rate $\omega/\gamma \sim 2\%$. Using SparSpec, we obtain $\omega/\gamma \sim 1.7\%$ for the $n = 5$, $\omega/\gamma \sim 0.9\%$ for the $n = 7$ and $\omega/\gamma \sim 0.45\%$ for the $n = 11$ modes, respectively, using a CPU time for this calculation of around 2ms for each time point in the data. Had we used an SVD decomposition algorithm similar to the one presented in [17], requiring a combinatorial exploration of all possible solutions and an a-posteriori thresholding scheme to determine the correct ones, we would have found very similar results as far as $n$ and $\omega/\gamma$, but now using a CPU time usually in excess of 150ms for each individual time point. The incredible speed and accuracy of the SparSpec algorithm has made it possible to deploy it in our real-time plant control software, which allows real-time detection and tracking of individual mode numbers within a CPU-time of $<600\mu$s for each 1ms real-time clock cycle. This now allows a detailed quantitative analysis of these measurements, as mode numbers can be directly separated in real-time and individually tracked to measure the changes in the mode frequency and damping rate during the evolution of the plasma background.

Regarding error analysis, the digital synchronous detection system allows for a very precise determination of the mode’s frequency, as the bandwidth of the optical phase-lock loop used in the hardware is such that the frequency is known to within less than 50Hz. The accuracy of the absolute amplitude, mode number and damping rate of the synchronously detected magnetic perturbations is affected by the presence of electrical (white) noise in the measurements, and particularly any DC offset, and by the errors on the calibration of the complex raw signal from the different pick-up (Mironov-type) coils and engineering signals (antenna current and voltages) used for the analysis. We estimate that the absolute value for the plasma response $|\delta B_r(\omega)|$ to the antenna-driven radial magnetic field $|B_r|$ is correct within a factor 2 for all frequencies, and as such it should be noted that this somewhat large uncertainty has only a minimal effect on the accuracy of the damping rate data, which are extracted from fitting $\delta B_r(\omega)$ in the complex plane. For the accuracy on the determination of the mode numbers one has to consider as well the possible statistical and systematic errors due to the numerical algorithm used to extract such data. Given the constraints imposed by SparSpec, the spectral window of the magnetic sensors used for this analysis, and the extensive tests carried out on real and simulated JET data, we have ascertained that the toroidal mode numbers can be determined exactly (i.e. $n = n \pm 0$) up to $|n| \sim 10\sim 12$ for modes whose (normalized) amplitude is at least 35% of the maximum amplitude in the spectrum (depending on the amount of noise in the measurements), with an error that can be up to $\pm 2$ for higher mode numbers and/or lower amplitudes modes. Finally, for the accuracy of the damping rate, one has to add the errors from the algorithm used to fit the measurements, leading in total to an uncertainty on $\omega/\gamma$ estimated to be around 30% for the typical cases that we consider in our analysis.

3. THE DEPENDENCE OF THE DAMPING RATE OF $n = 3$ AND $n = 7$ TAES ON THE EDGE ELONGATION
As many different damping mechanisms have been theoretically proposed for AEs, systematic experimental studies are needed to obtain the dependence of the AE damping rate on the background
plasma parameters. With this approach, one can then find the plasma parameters that are more important for the stability of AEs, and hence determine and quantify with direct comparisons with model calculations the dominant damping mechanisms in current devices. From then on, not only it is relatively simple to extrapolate with confidence to future devices such as ITER when considering the same experimental conditions, but one becomes also able to devise those particular experimental conditions where usually less important damping mechanisms may become dominant, which is in fact what is theoretically foreseen to occur for ITER. As an example of this line of experiments, the ion and electron Landau damping and the radiative damping mechanisms contribute very little in JET to the damping rate of low-n AEs in ohmic plasmas [18, 19], but these are expected to be the dominant damping mechanisms in ITER for AEs with \( n \sim 5-10 \) due to the much higher plasma temperature.

Conversely, in JET the edge plasma shape and magnetic shear has been found experimentally [14, 20] and theoretically [21, 22] to be a key ingredient for increasing the damping of both stable, antenna-driven low-n (\( n = 1, n = 2 \)) and unstable, fast-ion driven medium-n (\( n \sim 3-10 \)) AEs. This has motivated previous experimental studies on the Alcator C-mod tokamak where, contrary to the JET results for \( n = 1 \) and \( n = 2 \) TAEs, it was found that the damping rate of an \( n = 6 \) TAE remains essentially invariant when the average edge elongation \( \delta_{95} \) is scanned in the range 0.3<\( \delta_{95} <0.7 \), which is also associated to a similar variation in the edge elongation [23].

With the new set of JET antennas, it has now become possible to repeat the previous low-n measurements for modes with \( |n| \sim 3-12 \). In this respect, the capability of a real-time detection and tracking of the individual \( n \)-components in the antenna driven spectrum constitutes an invaluable tool, which is unique to JET and is lacking in other devices, to provide an accurate testing for the code predictions, as it is paramount that the same mode be measured throughout the parameter scan. We show here the measurement of the damping rate for an \( n = 3 \) (JET Pulse No: 77788, fig.5a) and an \( n = 7 \) (JET Pulse No: 77790, fig.5b) TAE as function of the edge elongation \( \delta_{95} \) for ohmic plasmas, where the only additional heating was provided by 200ms-long Neutral Beam blips which were used for diagnostic purposes to measure the ion temperature, rotation and safety factor profiles. Together with the main plasma parameters, in fig.5(a,b) we also show some illustrative examples of the \( n = 3 \) and \( n = 7 \) resonances measured in the synchronously detected \( \delta B_{r} \) spectrum and their fit in the complex plane. The very good agreement between the measured \( \delta B_{r} \) data and the fit is a clear indication of the accuracy of the damping rate measurements for each individual mode number.

The different damping rate measurements for the \( n = 3 \) and \( n = 7 \) TAEs obtained during these two discharges are shown separately in fig.6(a,b). We note an almost linear increase in the damping rate as function of the edge elongation, \( \omega/\gamma \propto \kappa_{95} \), for these two modes, which is essentially in very good agreement with the previous JET results for the \( n = 1 \) and the \( n = 2 \) TAEs, but in clear contradiction with the measurements made for an \( n = 6 \) mode in Alcator C-mod. For otherwise very similar background plasma parameters, the damping rate for the \( n = 3 \) TAE increases from \( \omega/\gamma \sim 0.3\% \) at \( \kappa_{95} \sim 1.33 \) to \( \omega/\gamma >5\% \) for \( \kappa_{95} >1.5 \), hence a factor 20 increase for a variation in the edge elongation of \( \sim 0.17 \). For the \( n=7 \) TAE, this increase in the damping rate is not as sharp, as \( \omega/\gamma \sim 4\% \).
for $\kappa_{95} \sim 1.35$ and $\omega/\gamma \sim 6.5\%$ for $\kappa_{95} \sim 1.4$, i.e. a 60% increase over a variation of $\sim 0.05$ in the edge elongation, whereas the corresponding values for the $n=3$ mode are $\omega/\gamma \sim 1.1-1.3\%$ for $\kappa_{95} \sim 1.35$ and $\omega/\gamma \sim 1.5\%$ for $\kappa_{95} \sim 1.4$, i.e. almost no (or a much smaller) variation in the measured damping rate. This result is particularly important because it confirms that the same damping mechanism acting upon global, low-$n$ modes, in fact plays a substantial role also for the stability of more core-localised medium-$n$ TAEs, opening up interesting perspectives for real-time control of these modes.

4. MODEL CALCULATION OF THE DAMPING RATE OF $n = 3$ TAEs USING THE LEMAN CODE

Model calculation of the damping rate of $n=3$ TAEs have been performed using the LEMan code [9, 10]. LEMan is a full-wave code that has been designed to compute propagation of low-frequency waves in the Alfvén and Ion Cyclotron ranges of frequencies. A warm-plasma formulation is now implemented in the code via the determination of a new dielectric tensor. This is based on the linearization of the Vlasov equation and on the Finite Larmor Radius (FLR) expansion where only terms of the order “0” are retained. This is however sufficient to model the Kinetic Alfvén Wave and Landau damping and thus to evaluate the absorption of global modes like TAEs. The choice to retain only these terms is motivated by the special attention that has been taken on the exact evaluation of the parallel wave vector. The latter has a strong impact on the wave propagation and the method required to retain its exact expression leads to a large increase of the computational resources. As LEMan deals with three-dimensional geometries, the equilibrium is computed with the VMEC code [25]. It is then mapped into the Boozer coordinates with TERPSICHORE [25]. This choice has the advantage of providing a sufficiently simple expression for the magnetic field while reducing the range of poloidal Fourier modes coupled to the antenna in the Alfvén domain of frequencies. The TERPSICHORE code has however not been yet adapted to geometries with updown asymmetry. This implies that experimental measurements containing such asymmetry, which is inevitable in the presence (or when moving closer to the formation) of an X-point, even in the core plasma, cannot be handled exactly with the LEMan code.

From a numerical point of view, LEMan solves the Maxwell equations in term of potentials. This allows avoiding the so-called numerical pollution that can lead to unphysical results. A weak Galerkin form has been employed to solve the set of equations obtained. The discretization is made in terms of Hermite cubics finite elements in the radial direction as well as Fourier decomposition along the toroidal and poloidal angles. For the simulations presented further, all poloidal mode numbers ($m$) from $m = -5$ to $m = 22$ have been retained. As it is hard to know which ones of them are driven in the experiment (the antennas are all located at one single position in the poloidal plane), the modelled antenna in LEMan excites equally the whole range of harmonics.

The comparison with the experimental measurements is performed for Pulse No: 77788 for the $n = 3$ TAE at the following three time steps: $t = 6.113\text{sec}$, $t = 10.096\text{sec}$ and $t = 14.1099\text{sec}$. The equilibrium is reconstructed using the data fitted by the EFIT code [26]. It has to be mentioned that
in the LEMan code, the radial coordinate $s$ ($s=0$ in the centre and $s=1$ at the border of the plasma) is defined as the square root of the toroidal flux instead of the poloidal flux as in most of the usual 2D codes. Hence, all data have been converted to this radial label, also the one employed in the VMEC and TERPSICHERE codes, and the simulation results are displayed using this representation. The results of these calculations are shown in fig.7a,b ($t=6.113\sec$), fig.8a,b ($t=10.096\sec$) and fig.9a,b ($t=14.109\sec$) for the three time points indicated above, respectively. Each figure is presented in two separate subplots (a,b): on the left frame (subplot-a), we show the antenna-plasma loading and the different branches of the continuum spectrum around the frequency of the TAE gap; on the right frame (subplot-b), we show the integrated power absorption and the Eigenfuction for different poloidal components for the parallel vector potential $A_{||}$.

The gap formed by toroidicity is easily recognizable in the left frame of each figure: several peaks are visible in the antenna-plasma loading at different frequency, and some of these peaks are located inside the TAE gap. When performing the experimental measurements, it is impossible to observe all of them, as the “real” antenna system excites and detects only one frequency at any one given time. It is straightforward to establish to which of those simulated peaks corresponds the measured TAEs, when considering the frequency range where the antenna sweep is undertaken. For example, in the case $t=6.113\sec$ (shown in fig.7), the experimental scan is essentially located between 180kHz and 210kHz. The TAE is then to be found in this domain. The only peak satisfying this condition in fig.7a has a frequency of 195kHz, which agrees quite well with the experimental data. Once the frequency of the calculated mode has been matched to the experimental value, the damping is then simply evaluated by determining the width of the loading peak in the LEMan frequency scan, which gives $\omega/\gamma_{\text{CALC}} = 1.1\%$, in good agreement with the experimental value of $\omega/\gamma_{\text{MEAS}} = 1.2\%$. In fig.7b, the Eigenfunction of this $n=3$ TAE is represented: from $s=0.4$ outward the characteristic structure of a global mode is recognizable, as it involves a large quantity of Fourier poloidal components from $m=5$ to $m=14$. In the region closer to the magnetic axis, the Alfvén branches $m=3$ to $m=5$ intersect with the continuum and contribute considerably to the power absorption. This is obvious when looking in details at the bottom frame graph in fig.7b, which represents the integrated power deposited from the centre to the label $s$ under consideration. Thus the steeper is the trend, the more the absorption is located at this point: for this particular case, most of the absorption occurs around $s\sim0.15$, i.e. it is dominated by the two $m=3$ and $m=4$ poloidal harmonics.

For the second situation at $t=10.096\sec$ (shown in fig.8), a TAE has been identified in the LEMan code at $f=171kHz$, which can be observed in the plasma response scan displayed in fig.8a. The calculated value of the frequency is $\sim4.5\%$ below the experimentally determined frequency at $t=10.073\sec$. A simple explanation for this difference is related to the neutral beam blip injected from $t=10\sec$ for diagnostics purpose. Between the time at which the $n=3$ mode is measured and that at which the background plasma profiles are determined (used as input for the LEMan code), the density has slightly increased. This implies a lower value for the simulated TAE frequency and explains partly the difference between the experimental and the numerical results. Concerning the
damping, good agreement has again been found between the measurement and the LEMan calculations, as in the first case, with $\omega/\gamma_{\text{CALC}} \sim \omega/\gamma_{\text{MEAS}} \sim 2\%$. As shown in fig.8b, the power absorption and Eigenfunction (computed using all poloidal harmonics between $m = 2$ and $m = 15$) are very similar to those shown in fig.7b for the first time point, with an additional power absorption step around $s \sim 0.1$.

Finally, for the simulation at $t = 14.109$sec, a peak corresponding to an $n = 3$ TAE is identified at $f = 179$kHz, again in good agreement with the experimental measurement of the mode frequency. On the other hand, this is however not the case for the calculated damping rate $\omega/\gamma_{\text{CALC}} = 2.0\%$, which is much smaller than the measured value, $\omega/\gamma_{\text{MEAS}} \sim 5.2\%$. In order to give an explanation for this clear discrepancy, fig10 displays both the original experimental equilibrium and the one that has been used for the simulations with the LEMan code by retaining the up-down symmetry, which will be discussed later. To complete the analysis for this time step, and despite the disagreement between the measured and calculated damping rate, it is still interesting to investigate the Eigenfunction and power absorption, displayed in fig.9b. The global mode is constituted of a more important number of harmonics than before (from $m = 2$ to $m = 20$). This is due the $q$-profile having changed significantly compared to the two first situations, particularly towards the plasma edge: in fact, $q$ now reaches $q(s = 1) = 5.44$ at the last closed flux surface. The integrated power in fig.9b shows also that the most of absorption is now located towards the plasma edge, for $s > 0.7$, implying that the proportion of the absorption located in the TAE continuum (at the plasma edge) compared to the part related to mode conversion with the harmonics $m = 3$ to $m = 5$ (in the plasma core) is much more important than at $t = 6.113$sec and $t = 10.096$sec.

Coming back now to the disagreement between the measured and computed damping rate for the third simulation, we note from fig.10 that at $t = 6.113$sec (fig7, first case) and $t = 10.096$sec (fig8, second case), the up-down symmetry is well satisfied and the magnetic configurations used in the simulation are very close to the experiment. This is no longer the case in the third simulation at $t = 14.109$sec (fig9), where a lower X-point appear because of the much higher edge elongation. The simulation cannot then be performed on the “real” equilibrium (i.e. as given by EFIT) and this explains why the results are imprecise in this particular case.

5. OUTLOOK AND FUTURE WORK

The first measurements of antenna-driven AEs with toroidal mode number in the range $|n| \sim 3$-15 have convincingly demonstrated that many of such modes exist at very similar frequencies in the plasma rest frame, such that their frequency separation is less than the modes’ half-width at halfmaximum, i.e. the damping. This has prompted the development of a sophisticated real-time mode number detection and separation algorithm for such a frequency-degenerated spectrum, which is based on the sparse representation of signals. Routine measurements of the damping rate for low and medium- to high-$n$ AEs have now been obtained for various JET operating regimes, with realtime tracking of the driven resonances providing tens of damping rate data for each individual
mode number on a single discharge. This result has been obtained using compact antennas with a small effective area, furthermore located rather far away from the plasma edge, at a typical distance in excess of 60mm from the last closed flux surface. This is therefore a very promising technical result in view of a possible use of compact active antennas in ITER for burn control applications.

The first quantitative analysis of the damping rate measurements obtained for medium-n TAEs, \(n = 3\) and \(n = 7\), has confirmed the experimental scaling of an increase in \(\omega/\gamma\) as the edge elongation (hence the edge magnetic shear) is increased. This measured scaling is in agreement not only with previous measurements in JET for low-n TAEs, but also with theoretical estimates based on the mode conversion of TAEs to kinetic Alfvén Waves at the plasma edge. However, this contradicts results obtained in the Alcator C-mod tokamak, and one possible reason for this discrepancy is the absence in this device of real-time tracking or post-pulse discrimination of the individual mode numbers making up a degenerate frequency spectrum.

The first quantitative measurements of the damping rate for individual mode numbers made in JET have now been made available for detailed comparisons with theory and models. A first benchmark has been performed using the LEMan code for \(n = 3\) TAEs, which has conclusively demonstrated the importance of retaining the up/down asymmetry in the plasma poloidal cross section in order to be able to quantitatively reproduce the measured scaling of the damping rate for this mode as function of the edge elongation. For such cases where the plasma poloidal cross section is sufficiently up/down symmetric, the LEMan results for the more frequency and damping rate are in very good agreement with the measurements, and also demonstrate that a large number of poloidal harmonics should be used so as to reproduce correctly the measured \(\omega/\gamma\) Further benchmark work with other codes is the subject of ongoing work within the ITPA work-programme.

Together with the ability of detecting and correctly separating in real-time individual mode numbers in a multi-component stable AE spectrum, the confirmation, first reported in this work, of the low-n scaling of \(\omega/\gamma = f(\kappa_{95})\) for the more interesting medium-n TAEs, as these are fast ion (and specifically \(\alpha\)’s) driven modes, opens the door for the implementation of a closed real-time loop for the control of the TAE stability in burning plasmas, such as the regimes foreseen for ITER. The damping rate measurements are already available in the JET real-time signal server, and their real-time estimate has been shown to be in very good agreement with the result obtained with a more detailed postpulse analysis. To simulate in JET plasmas the destabilising contribution of the \(\alpha\)-particle drive, one can use the anisotropic MeV energy ions produced by Ion Cyclotron Frequency Heating or the mostly-isotropic, but lower energy, \(\sim 150\)keV Neutral Beam ions. As sketched in fig.11, when the mode’s damping rate reduces too much, the edge elongation can be increased via a small increase in the current (i.e. the actuator) of the shaping coils, so as to bring the plasma back to a situation where it is further away from the marginal stability limit.

On the more technical aspects, an upgrade in the excitation system is now planned for the near future, so as to be able to drive independently all the antennas and at higher power. This could create further burning plasma control situations, where the antennas are used to excite modes at
higher amplitude to cause a controlled redistribution of the fast particles and prevent an excessive peaking of their pressure gradient. This would then reduce the risk of a more violent fast ion redistribution, which could to a total loss of ignition and plasma confinement.

ACKNOWLEDGEMENTS
This work has been conducted under the European Fusion Development Agreement. The Authors would like to thank the various members of the CRPP, MIT and JET staff that have contributed to the design, installation, commissioning and routine operation of the new TAE antenna system. The Authors would also like to acknowledge the contribution of Alex Klein (formerly at MIT) for his work on the initial development of the real-time application of the SparSpec code.

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[10]. N. Mellet, PhD Thesis; to be submitted for publication.

Figure 1: View of one group of four AE antennas as installed in vessel during June 2005.
Figure 2a: The total radial nominal magnetic field driven in vacuum by four neighbouring antennas with the same (++++) phasing and current $I_{ANT} = 1A$; the value $R = 4.1m$ indicates the antenna position along the major radius coordinate, and $R = 2.98m$ is the geometrical centre of the JET tokamak. Note the rapid exponential decay of the antenna-driven radial magnetic field as function of the distance from the antenna, so that $\max(|B_r|) \sim 0.01G$ in the plasma core even at maximum antenna current.

Figure 2b: The total radial nominal magnetic field driven in vacuum by four neighbouring antennas with alternating (+-+-) phasing and current $I_{ANT} = 1A$; the value $R = 4.1m$ indicates the antenna position along the major radius coordinate, $R = 2.98m$ being the geometrical centre of the JET tokamak. The exponential decay of the antenna-driven radial magnetic field is even faster for this antenna configuration than for the ++++ case shown in fig2a, as in fact $\max(|B_r|) \sim 0.005G$ in the plasma core even at maximum antenna current.

Figure 2c: The radial dependence of the antenna driven magnetic field (for a nominal antenna current $I_{ANT} = 1A$) versus the minor radius coordinate for different components in the toroidal spectrum as produced by different combination of antennas being excited. Note that by suitable selection of the antenna phasing, we can preferentially drive odd or even $n$-components (as shown in the central frame): the inexact cancellation of the unwanted components is due to the antennas not being located exactly at the major radius position. Finally, as shown in the rightmost frame, note that even when higher-$n$ components are larger at the plasma edge, they become less important closer to the magnetic axis as their exponential fall with the distance from the antennas is faster than that for lower $n$-modes.
Figure 2d: Two examples of the toroidal spectrum of the antenna driven radial magnetic field, calculated using the actual antenna current $I_{\text{ANT}}$ driven in each discharge and the plasma geometry: using different antenna phasing, we can drive predominantly (in this odd) $n<5$ or $n\sim6-15$ modes. Note again the non-exact cancellation of the unwanted ($n = \text{even}$) spectrum components.

![Graph showing driven n-spectrum](image)

**Figure 3:** The antenna-plasma coupling calculated using the LION code on the same real JET magnetic equilibrium (Pulse No: 33157) for different versions of the antenna designs, labelled with their number and position of its turns; with the current design (18 turns, with turns located between 47mm and 183mm from the last closed flux surface). These calculations show that a similar coupling should be obtained for an $n = 5$ TAE to that achieved for an $n = 2$ TAE with much bigger saddle coils, which had about $80\times$ the effective area of the new antennas. These modelling results have been broadly verified using direct measurements of the antenna-drive radial magnetic field via in-vessel pick-up coils for a variety of plasma configurations.
Figure 4a: spectrogram of the magnetic perturbation measured for the JET Pulse No: 77308: three different bands where modes are excited by the AE antennas are observed, but it is only with the SparSpec analysis shown in fig 4b that we can determine precisely the mode numbers.

Figure 4b: SparSpec analysis of the synchronously detected data corresponding to the spectrogram shown in fig 4a: we find that three separate $n=5$, $n=7$ and $n=11$ coexist in the lowest frequency band around 160kHz.

Figure 5a: Overview of the main background plasma parameters for the JET Pulse No: 77788 where an $n=3$ TAE was detected and its evolution tracked in real-time; we also show a few illustrative examples of the measured $\delta Br$ and its complex-plane fit.
Figure 5b: Overview of the main background plasma parameters for the JET Pulse No: 77790 where an n = 7 TAE was detected and its evolution tracked in real-time; we also show a few illustrative examples of the measured δBr and its complex-plane fit.

Figure 6a: The scaling of the measured damping rate for the n=3 TAE as function of the edge elongation κ95 in the two similar JET Pulse No’s: 77788 and 77790.

Figure 6b: The scaling of the measured damping rate for the n=7 TAE as function of the edge elongation κ95 in the two similar JET Pulse No’s: 77788 and 77790.
Figure 7 (a,b): Results of the LEMan simulation at $t=6.113s$ with the parameters on the axis $B=2.7T$, $n_{e0}=2.4\times10^{19}m^{-3}$ and $T_{e0}=T_{i0}=2.6keV$. In fig7a (top) we shown the frequency scan of the plasma response to the antenna drive (left frame) and the Alfvén branches spectrum computed using the dispersion relation (right frame). In fig7b we plot the Eigenfunction of the parallel component of the vector potential (bottom frame) and the integral of the power deposition profile (top frame) for the calculated mode at $f=195kHz$ (which corresponds to the measurement).

Figure 8 (a,b): Results of the LEMan simulation at $t=10.096s$ with the parameters on the axis $B=2.7T$, $n_{e0}=2.6\times10^{19}m^{-3}$ and $T_{e0}=T_{i0}=2.4keV$. The frames are as in fig7(a,b): the calculated mode at $f=171kHz$ matches the measured frequency and is used for the calculation of the power absorption.
Figure 9: (a, b): Results of the LEMan simulation at t=14.109s with the parameters on the axis B = 2.7T, n_e0 = 2.4 × 10^{19} m^{-3} and T_{e0} = T_{i0} = 2.6 keV. The frames are as in fig7(a,b): the calculated mode at f = 179kHz matches the measured frequency and is used for the calculation of the power absorption.

Figure 10: Outer surface of the experimental equilibrium obtained with EFIT (dashed line) compared with the outer surface of the up-down symmetric equilibrium used as input in VMEC (solid line) at t=6.113sec (left frame), t=10.096sec (middle frame) and t=14.109sec right frame). Note the clear difference between these two equilibria when the plasma configuration develops an up/down asymmetry because of the increasing elongation.
Figure11: Example of a possible real-time control scheme for TAEs in burning plasmas. First, a stable mode is detected in real-time, and then its damping rate decreases (as measured by the frequency width of the antenna-driven resonance) as the plasma evolves towards the marginal stability limit. At this point, a feedback command is sent to some suitable actuators, in our case the edge shaping coils, with a request to increase the current in these coils, so as to increase the edge elongation by a very limited amount, of the order of 5-15% as from the measurements reported here. With this scheme, we then expect the damping rate to revert back to a value sufficiently far away from the marginal stability limit.