Current, Position and Shape Control in Tokamaks
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Abstract

The need of achieving always better performance in present and future tokamak devices has pushed plasma control to gain more and more importance in tokamak engineering. When high performance and robustness are required, it is essential to adopt a model-based approach to design a control system. In this paper we introduce the basics of plasma current, shape position control in tokamaks. As example, the plasma magnetic control systems of the JET tokamak will be presented, together with an approach proposed for plasma current, position and shape control at the ITER tokamak.

I. INTRODUCTION

The search for new resources has become crucial in this time of increasing energy demand. Research in nuclear fusion field tries to give an answer to this request. One possible approach for nuclear fusion on earth is the magnetic confinement of a plasma in suitable devices. Among the various possible configurations, the most promising approach has proved to be the tokamak [1].

In a tokamak reactor, plasma is formed into a vacuum chamber (the vessel shown in Fig. 1), and several magnetic fields have to be applied to confine the plasma. The dominant one, the toroidal magnetic field, is generated by a set of coils named Toroidal Field (TF) coils. However, a plasma placed in such a field cannot come to an equilibrium force balance. Indeed the pressure of a toroidal plasma would cause it to expand and the toroidal magnetic field is unable to provide a balancing force. The result would be a very rapid loss of the whole plasma. For this reason an additional magnetic field component should be added to confine the plasma. In the tokamak configuration this difficulty is overcome by passing a toroidal current through the plasma itself. By using the plasma as secondary winding of a transformer, a plasma current $I_p$ and the related poloidal magnetic field are generated. The combined (toroidal and poloidal) magnetic field is helical. Another component is added to the plasma generated poloidal field by means of the Poloidal Field (PF) coils shown in Fig. 1. This additional component is used to both achieve the desired plasma configuration and to control the plasma shape and position[33].

The ITER tokamak [2] is the next step toward the realization of electricity-producing
fusion power plants. Its main goal is to obtain plasma burning condition and produce about 500 MW of fusion generate-power for more than 400 s [3]. To estimate the effort required to build ITER, it may help noting that its major radius is twice the one of the Joint European Torus (JET) [4], which is the today’s world largest fusion reactor. Furthermore the plasma current in ITER will be of about 15 MA, which is more than twice the maximum current obtained during JET operation in the last 30 years.

The need of achieving always better performance in present and future tokamak devices has pushed plasma control to gain more and more importance in tokamak engineering (see [5] and special issues [6] and [7]). High performance in tokamaks is achieved by plasmas with elongated poloidal cross section. Since such elongated plasmas are vertically unstable, position control is clearly an essential feature of all machines. Beyond this, a strong motivation to improve plasma control is that, in order to obtain the best performance out of a device, it is always necessary to maximize the plasma volume within the available space; hence, the ability to control the shape of the plasma while ensuring good clearance between the plasma and the facing components is an essential feature of any plasma position and shape control system. When high performance and robustness are required, it is essential to adopt a model-based approach to design a control system [5, 8].

This paper is structured as follows: first the plasma linearized model is introduced. This model can be effectively used to design plasma current, shape and position control systems. Afterwards, in Section III, the basic magnetic plasma problems are presented, i.e. the plasma current, position and shape control problems. Design and implementation issues of the plasma magnetic control systems at the JET tokamak are discussed in Section IV, while in Section V a possible plasma current, position and shape control approach for the ITER tokamak is presented.

II. PLASMA MODELING

This section introduces a axisymmetric plasma linearized model that can be effectively used to design magnetic control. For a comprehensive treatment about plasma modeling for magnetic control, refer to [5, Ch. 2] and [8].

Magnetohydrodynamics (MHD [9, Ch. 11]) provides a macroscopic dynamical description
of an electrically conducting fluid in the presence of magnetic fields. Starting from MHD theory, behavior of the system consisting of the plasma, the surrounding passive structures, and the external PF coils, can be described by a set of nonlinear partial differential equations (PDEs).

Although it has recently been proven that neglecting plasma mass may lead to erroneous conclusion when designing the vertical stabilization system [10], in this paper we assume that the inertial effects can be neglected at the time scale of interest, since plasma mass density is low. Neglecting the plasma mass, the system dynamic can be regarded as a sequence of equilibria. Moreover, it is possible to introduce a number of simplifying assumptions, such as axisymmetry of the tokamak. Given these assumptions and using approximate numerical methods, the following nonlinear lumped parameters approximation of the PDEs model is obtained:

\[
\frac{d}{dt} \left[ M(y(t), \beta_p(t), l_i(t)) I(t) \right] + RI(t) = U(t),
\]
\[
y(t) = Y(I(t), \beta_p(t), l_i(t)),
\]

where:

- \( y(t) \) are the output to be controlled;
- \( I(t) = [I^T_{PF}(t) \ I^T_e(t) \ I^T_p(t)]^T \) is the currents vector, which includes the currents in the active coils \( I_{PF}(t) \), the eddy currents in the passive structures \( I_e(t) \), and the plasma current \( I_p(t) \);
- \( U(t) = [U^T_{PF}(t) \ 0^T \ 0]^T \) is the input voltages vector;
- \( M(\cdot) \) is the mutual inductance nonlinear function;
- \( R \) is the resistance matrix;
- \( Y(\cdot) \) is the output non linear function.

The poloidal beta \( \beta_p(t) \) and the internal inductance \( l_i(t) \) are measures of the plasma internal distributions of pressure and current, respectively. These parameters are used to take into account the changing in plasma response due to the changing of the plasma internal profiles.
Starting from (1), the following plasma linearized state space model \[11, 12\] can be easily obtained:

\[
\begin{align*}
\delta \dot{x}(t) &= A \delta x(t) + B \delta u(t) + E \delta \dot{w}(t), \\
\delta y(t) &= C \delta I_{PF}(t) + F \delta \dot{w}(t),
\end{align*}
\]

(2a) (2b)

where:

- \(A\), \(B\), \(E\), \(C\) and \(F\) are the model matrices;
- \(\delta x(t) = \begin{bmatrix} \delta I_{PF}^T(t) & \delta I_{e}^T(t) & \delta I_{p}(t) \end{bmatrix}^T\) is the state space vector;
- \(\delta u(t) = \begin{bmatrix} \delta U_{PF}^T(t) & 0^T & 0 \end{bmatrix}^T\) are the input voltages variations;
- \(\delta w(t) = \begin{bmatrix} \delta \beta_p(t) & \delta l_i(t) \end{bmatrix}^T\) are the \(\beta_p\) and \(l_i\) variations;
- \(\delta y(t)\) are the output variations.

The model (2) relates the variations of the PF currents to the variations of the outputs around a given equilibrium. Moreover, as far as the plasma magnetic control is concerned, the \(\delta \beta_p(t)\) and \(\delta l_i(t)\) variations, together with the plasma current variation, can be regarded as disturbances.

Given the model (2), different outputs can be chosen, for example:

- fluxes and fields at the positions where the magnetic sensors are located;
- currents in the active and passive circuits;
- plasma radial and vertical position;
- geometrical descriptors describing the plasma shape (gaps, x-point and strike points positions, see Fig. 2).

### III. PLASMA AXISYMMETRIC CONTROL PROBLEMS

The three fundamental axisymmetric magnetic control problems in tokamaks are presented in this section, namely the Vertical Stabilization Problem (VSP), the Plasma Shape Control Problem (PSCP), and the Plasma Current Control Problem (PCCP).
A. Vertical Stabilization Problem

High performances in tokamaks are achieved by plasmas with elongated poloidal cross-section\cite{34} and magnetic X-point \cite{13, Tutorial 7}. Since such elongated plasmas are vertically unstable, position control is clearly an essential feature of those machines.

In order to illustrate the plasma vertical instability, let consider the simplified filamentary model depicted in Fig. 3, where two rings are kept fixed and in symmetric position with respect to the $r$ axis, while the third, which models the plasma, can freely move vertically. If the currents in the two fixed rings that model the active coils are equal, then the vertical position $z = 0$ is an equilibrium point for the system. Given the plasma current $I_p$ and the current in the active coils $I$, the following two cases must considered:

1. $\text{sgn}(I_p) \neq \text{sgn}(I)$ - the plasma poloidal cross-section is circular and the equilibrium is vertically stable (see Fig. 4);
2. $\text{sgn}(I_p) = \text{sgn}(I)$ - the plasma poloidal cross-section is elongated and the equilibrium is vertically unstable (see Fig. 5).

Hence, it turns out that elongated plasmas are vertically unstable. The plasma vertical instability reveals itself in the linearized model (2), by the presence of an unstable eigenvalue in the dynamic system matrix $A$.

The vertical instability growth time $\gamma$ is slowed down by the presence of the conducting structure surrounding the plasma. This permits to use a feedback control system to stabilize the plasma equilibrium.

The main objectives of the any Vertical Stabilization (VS) system are:

- to vertically stabilize elongated plasmas, in order to avoid disruptions;
- to counteract the effect of disturbances such as Edge Localized Modes (ELMs \cite{14}), fast disturbances modeled as Vertical displacement Events (VDEs \cite{15}), etc.

It is worth to notice that the VS control loop usually acts on a faster time-scale than the plasma shape control loop.
B. Plasma Shape Control Problem

The problem of controlling the plasma shape is probably the most understood and mature of all the control problems in a tokamak. The actuators are the PF coils (see Fig. 6), that produce the magnetic field acting on the plasma, while the controlled variables are a finite number of geometrical descriptors chosen to describe the plasma shape. An example of plasma boundary descriptors usually controlled at the JET tokamak are shown in Fig. 2.

The main objectives of any Shape Controller (SC) are:

- to precisely control the plasma boundary;
- to counteract the effect of disturbances (in terms of $\beta_p$ and $l_i$ variations);
- to manage the saturation of the actuators, i.e. the current saturations in the PF coils [16].

Taking into account the presence of the VS system, the two control schemes shown in Fig. 7 are considered in this paper for plasma shape control. In the control scheme of Fig. 7(a) the SC generates voltage requests for the power supplies that feed the PF circuits. In this case both the VS and the SC generates voltage requests. Furthermore, adopting this scheme, the scenario voltages, i.e. the voltages needed to achieve the desired target in terms of plasma current and shape, must be specified in feedforward in addition to the control voltages, i.e. the voltages generated to counteract the disturbances.

Since the scenario is usually specified in terms of current waveforms to be fed by the power supplies into the PF circuits, it can be convenient to resort to the control scheme in Fig. 7(b), where a current controller is designed to control the current in each PF circuits. In this case the scenario is specified in terms of feedforward currents, and also the SC generates current requests.

It is important to remark that plasma shape control and vertical stabilization are usually performed on two different time scales. Indeed:

- for the JET tokamak, the time constant of the unstable mode is about 2 ms, while the settling time of the SC is about 0.7 s;
- for the ITER tokamak the envisaged time constant of the unstable mode is about 100 ms, while the settling time of the SC can vary between 15 and 25 s.
C. Plasma Current Control Problem

Plasma current is usually controlled by using the current in the PF coils. Since there is a sharing of the actuators, the problem of tracking the plasma current is often considered simultaneously with the shape control problem. However, shape control and plasma current control are compatible, since a magnetic flux that is spatially uniform across the plasma (but with a desired temporal behavior) can be used to drive the current without affecting the plasma shape.

IV. PLASMA MAGNETIC CONTROL AT THE JET TOKAMAK

This section describes both the SC and the VS system currently used at the JET tokamak.

A. The Poloidal Field Coils System at JET

Fig. 6 shows a poloidal cross-section of the JET tokamak where the PF coils are shown as red squares. These coils are linked together into 10 circuits driven by independent power supplies, named $P1$, $P4$, $IMB$, $SHA$, $PFX$, $D1$, $D2$, $D3$, $D4$ and $FRFA$.

The $P1$ circuit enables both the plasma inductive formation and the control of the plasma current. The current in the $FRFA$ circuit is driven by the VS system [17], while the remaining PF circuits are controlled either by the SC or by the eXtreme Shape Controller (XSC) to perform both plasma current and shape control.

B. The JET Shape Controller

The JET SC drives the current into the PF circuits shown in Fig. 6, and it was conceived as the solution to the shape control problem for the entire discharge.

During the plasma formation process, the SC controls the currents in PF circuits so that they track a set of pre-programmed waveforms. These waveforms have been empirically shown to give a successful breakdown. Afterwards a small plasma column is formed and slowly expands to fill the vessel volume. In this phase, since it is difficult to calculate the plasma shape precisely, the SC controls only the plasma current and the radial position. Depending on the experiment, different aspects of the shape become more important. The
main experimental phase typically starts after the plasma becomes bounded by a separatrix, and the control is switched to the *geometrical descriptors* which specify the plasma boundary (gaps, strike points and X-point position shown in Fig. 2).

The SC gives the possibility of controlling simultaneously up to six geometrical descriptors. This limitation has been overcome by the XSC which has been deployed at JET in 2003: with this new system it is possible to control, in a mean-square sense, more than 30 plasma shape descriptors using eight PF circuits, as it will be shown in Section IV C.

The person responsible for implementing an experiment (the *Session Leader*) programs the discharge dividing it in a number of *time windows*; in each time window, the PF circuits can be used in one of the following control modes:

**CURRENT CONTROL** the controlled variable is the current in the corresponding circuit; this is the case for instance of the breakdown phase;

**PLASMA CURRENT CONTROL** the controlled variable is the plasma current;

**GAP CONTROL** the controlled variable is a plasma boundary geometrical descriptor, typically a gap;

**BLOCKED** the current in the actuator is set to 0;

**FREE-WHEELEING** the voltage across the coil is set to 0.

The definition of the variables that can be selected in *gap control* mode, as well as the availability of the different control modes, differs for each PF circuit. For example the *plasma current control* mode is available only for the $P1$ circuit.

The SC design is based on a plasma linear model; only one model is used for all the cases of interest. The SC designers managed to obtain a controller that guarantees acceptable dynamic performance [18] in many different situations. This result has been achieved limiting the control bandwidth and tuning the controller parameters during the SC commissioning phase.

The SC design is based on the *plasmaless* model[35] of the PF coils system, which can
be written as

\[
\mathbf{U}_{PF}(t) = \begin{bmatrix}
L_1 & M_{12} & \cdots & M_{1N} \\
M_{12} & L_2 & \cdots & M_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
M_{1N} & M_{2N} & \cdots & L_N
\end{bmatrix}
\begin{bmatrix}
\dot{I}_{PF}(t) \\
\dot{I}_{PF}(t)
\end{bmatrix} + \begin{bmatrix}
R_1 & 0 & \cdots & 0 \\
0 & R_2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & R_N
\end{bmatrix}
\begin{bmatrix}
I_{PF}(t) \\
I_{PF}(t)
\end{bmatrix} = \mathbf{M} \mathbf{I}_{PF}(t) + \mathbf{R} \mathbf{I}_{PF}(t),
\]

(3)

The terms \(L_i\) represents the self-inductance of the \(i\)-th circuit, while \(M_{ij}\) is a mutual inductance term. \(R_i\) are the circuit resistances.

The control algorithm implemented by the SC is independent of the choice of control modes, and it is given by

\[
\mathbf{U}_{ref}(t) = \hat{\mathbf{R}} \mathbf{I}_{PF}(t) + \mathbf{K}(\mathbf{Y}_{ref}(t) - \mathbf{Y}(t)).
\]

(4)

In (4), \(\mathbf{U}_{ref}(t)\) are the voltage references to the amplifiers; \(\hat{\mathbf{R}}\) is an estimation of matrix \(\mathbf{R}\) in (3); \(\mathbf{K}\) is the gain matrix; \(\mathbf{Y}_{ref}(t)\) and \(\mathbf{Y}(t)\) are the reference signals and measurements, respectively, and depend on the controlled variables choice. A schematic of the SC is shown in Fig. 8.

The control matrix \(\mathbf{K}\) is constructed according to

\[
\mathbf{K} = \hat{\mathbf{M}} \mathbf{T}^{-1} \mathbf{A},
\]

(5)

where \(\hat{\mathbf{M}}\) is an estimation of the mutual inductance matrix in (3) and decouples the PF circuits, \(\mathbf{A}^{-1}\) is the diagonal matrix of desired time constants, and \(\mathbf{T}\) is the transformation matrix, which represents the linearized relation between the currents in the PF circuits and the measurements

\[
\mathbf{Y}(t) = \mathbf{T} \mathbf{I}_{PF}(t).
\]

(6)

Replacing relation (6) into (3), the open-loop system model reads

\[
\mathbf{M} \mathbf{T}^{-1} \dot{\mathbf{Y}}(t) = \mathbf{U}_{PF}(t) - \mathbf{R} \mathbf{I}_{PF}(t).
\]

(7)

Using equations (4)–(7), the closed-loop system can be written as

\[
\dot{\mathbf{Y}}(t) = \mathbf{A}(\mathbf{Y}_{ref}(t) - \mathbf{Y}(t)).
\]

(8)
The model in (8) is derived neglecting the delays introduced by the power supplies and by the diagnostics, and assuming $\hat{R} = R$ and $\hat{M} = M$.

It follows that for all the PF circuits set in current control mode, the corresponding $Y(t)$ entries are the PF currents, and the corresponding block in the matrix $T$ is the identity matrix.

After the plasma is formed, the plasma current circuit should be added in (3). The SC adopts a simplified model of the plasma current circuit, where the plasma resistance is neglected, and only the mutual inductance with the $P1$ circuit is retained. Since the plasma current distribution is a function of the magnetic fields, the mutual induction between $P1$ and the plasma depends on the currents in the circuits. However, this dependence is not relevant, and the following broadly valid linear model can be derived

$$\dot{I}_p(t) = -c \dot{I}_{P1}(t), \text{ with } c > 0,$$

where $I_p(t)$ and $I_{P1}(t)$ are the plasma current and the current in the $P1$ circuit, respectively. Model (9) is over-simplified since it does not take into account any plasma internal profile concise parameters, such as, for instance, $\beta_p$ and $l_i$. However this model is then used to determine the row corresponding to the plasma current in the $T$ matrix, when $P1$ is set in plasma current control mode.

C. The JET eXtreme Shape Controller

The XSC controls the whole plasma shape, specified as a set of geometrical descriptors (typically 32), calculating the PF coil current references. Its design [19] is based on the following plasma linearized state space model introduce in Section II. In particular, if $\delta g(t)$ are the $n_G(\leq 32)$ variations of the plasma shape descriptors, and $I_{P_{F,n}}(t)$ are the PF currents normalized to the equilibrium plasma current, then it is

$$\delta g(t) = C \delta I_{P_{F,n}}(t).$$

From (10) it follows that the plasma boundary descriptors have the same dynamic response of the PF currents.

The XSC design has been based on the $C$ matrix. It is worth to notice that the plant model is non-right-invertible, since $n_{PF} < n_G$, i.e. the number of independent control
variables is less than the number of outputs to regulate. For such a plant it is not possible
to track a generic set of references with zero steady-state error. Furthermore, given a generic
set of references, the best performance that can be achieved in steady-state is to control to
zero the error on \( n_{PF} \) linear combinations of geometrical descriptors. Control to zero such
an error is equivalent to minimize the following steady-state performance index (see [20]):

\[
J = \lim_{t \to +\infty} (\delta g_{ref} - \delta g(t))^T (\delta g_{ref} - \delta g(t)),
\]
(11)

where \( \delta g_{ref} \) are constant references to the geometrical descriptors.

Minimization of (11) can be attained using the singular value decomposition (SVD) of
the \( C \) matrix:

\[
C = U S V^T,
\]

where the matrix \( S \) contains the singular values, \( U \) and \( V \) are unitary matrices, that is

\[
U U^T = U^T U = I, \quad V V^T = V^T V = I.
\]

As a matter of fact, using the JET linearized models, it turned out that some singular
values (typically 2 or 3, depending on the configuration) are one order of magnitude smaller
than the others. This fact implies that minimizing the performance index (11) retaining
all the singular values result in a high control effort at steady-state, in terms of PF coil
currents. For this reason, the XSC achieves a trade-off condition, minimizing a modified
quadratic cost function that penalizes both the error on the controlled shape descriptors,
and the control effort. This is achieved controlling to zero the error only for the \( \bar{n} < n_{PF} \)
linear combination related to the largest 5 or 6 singular values [20].

A more sophisticated version of the XSC has then been implemented introducing weight
matrices both for the geometrical descriptors and for the PF coil currents.

As an example of plasma shape controlled by the XSC, the experimental results for the
JET pulse 68953 are shown Fig. 9. A comparison between the measured shape (solid trace)
and the desired reference (dashed trace) is reported in Fig. 9(a), while the time traces of six
plasma geometrical descriptors are shown in Fig. 9(b). Note that, even if the steady-state
error on the whole plasma boundary is not zero, the control error is kept less than one
centimeter on each plasma shape descriptor.

Fig. 10 shows the XSC control scheme. In this scheme, the current controller block is
the SC in which the \( P1 \) circuit is set in plasma current control mode and the remaining
eight PF circuits are set in current control mode (see Section IV.B). Thanks to this block, each PF circuit can be treated as an independent SISO (single-input-single-output) channel with a first order response. The low-pass filters in Fig. 10 set the time constants for all the PF circuits to the slowest one. Eventually, the gap controller block computes the \( \bar{n} \) linear combinations of the geometrical descriptors that are controlled to zero at steady-state.

To automate the controller design and validation phases, a set of Matlab/Simulink graphic applications, called XSC Tools [21], has been developed. Using these tools, new XSC scenarios can be easily prepared and validated via closed-loop simulations. Thanks to the model-based design approach, XSC functionalities have been easily extended in order to include strike-points sweeping [22] and plasma boundary flux control [23].

\[ D. \text{ The JET Vertical Stabilization system} \]

The block diagram of the JET VS system is shown in Fig. 11. The JET VS controls the plasma vertical velocity \( \dot{z}_p(t) \) rather than the position, since its main objective is to stabilize, and hence to stop, the plasma. The plasma vertical position \( z_p(t) \) is controlled on a slower timescale by the SC. In addition to plasma velocity, the VS system implements also a current control loop to keep the current in the actuator (the FRFA circuit shown in Fig. 6) as small as possible. In turns out that the control law implemented by the JET VS provides a proportional action on plasma velocity and a proportional-integral action on the actuator current, that is

\[
U_{FRFA_{ref}}(t) = G_v(t)\dot{z}_p(t) + G_I(t)(I_{FRFA_{ref}}(t) - I_{FRFA}(t)) + \frac{G_I(t)}{T_I} \int_0^t I_{FRFA_{ref}}(\tau) - I_{FRFA}(\tau) d\tau,
\]

where \( U_{FRFA_{ref}}(t) \) is the voltage reference for the power supply, while \( I_{FRFA_{ref}}(t) \) and \( I_{FRFA}(t) \) are the reference and the measurement of the current in the FRFA circuit, respectively. Since of of the controller objectives is to keep small the current in the actuator, typically \( I_{FRFA_{ref}}(t) \) is set equal to zero. \( G_v(t) \) and \( G_I(t) \) are the controller gains that are adapted during the discharge according to the variations of a number of plant parameters, such as the power supply switching frequency, its temperature, and the value of the current in the actuator.

In 2009, the Plasma Control Upgrade (PCU) project [24] has increased the capabilities of the JET VS system so as to meet the requirements for future operations at JET. In
particular, the ability of the VS system to recover from large ELMs, specially in the case of plasmas with large growth rate, has been enhanced. Within the PCU project, the design of the new VS system has included:

1. the design of the new power supply for the FRFA circuit [25];

2. the assessment of the best choice for the number of turns for the coils of the FRFA circuit;

3. the design of the new VS software, so as to deliver to the operator an high flexible architecture [26].

V. PLASMA CURRENT POSITION AND SHAPE CONTROL AT THE ITER TOKAMAK

This section briefly introduces a possible approach for plasma current, position and shape control for the ITER tokamak previously presented in [27]. The proposed approach combines the solutions previously presented in [28] and [19], and makes use of the in-vessel coils to vertically stabilize the plasma.

Although high performance are needed to reach the desired objectives, the design of the ITER plasma position and shape control system should take into account a number of constraints which are strongly related to the effective realization of the facility. In particular, while elongated plasmas with $\beta_p$ up to 1.9 and $l_i$ up to 1.2 are envisaged so as to guarantee the needed particle and energy confinement, the power available to control such plasmas is limited, since saturation levels of the actuators are present. Moreover the passive structures of ITER vessel introduce a not negligible delay on the control action, when the PF coils are used to perform the plasma vertical stabilization.

Recently, during the design review phase, it turned out that the high elongated and unstable plasmas needed for ITER operations can be hardly stabilized using the superconductive PF coils placed outside the tokamak vessel. For this reason it has been proposed to investigate the possibility of using copper in-vessel coils (see Fig. 1), so as to improve the best achievable performance of the vertical stabilization system [29]. It is worth to notice that these in-vessel coils cannot be superconductive, and there are thermal constraints that limit their operation.
The control architecture adopted in this case is the one depicted in Fig. 7(a), hence the SC generates voltage requests. In particular the in-vessel circuit as actuator for the VS system. Let the in-vessel $\delta U_{ic}(t)$ voltage be equal to

$$
\delta U_{ic}(t) = k_D \delta \dot{z}_p(t) + k_I \delta I_{ic}(t),
$$

(12)

where $\delta \dot{z}_p(t)$ and $\delta I_{ic}(t)$ are the plasma vertical velocity and the in-vessel current variations, respectively. Note that the in-vessel coils shown in Fig. 1 are connected in anti-series, that is the current in the upper coils flows in opposite way with respect to the current in the lower coils. Hence a single voltage and current are considered for the in-vessel circuit.

The two gains $k_D$ and $k_I$ of the VS system can be chosen so as to fix the closed-loop decay rate by solving the following Bilinear Matrix Inequalities [30, 31]. Furthermore, by exploiting a BMIs-based design approach, thermal constraints which limits the root-mean-square (rms) value of the current $I_{ic}$ can be taken into account (more details can be found in [15]).

Taking into account that the SC can act on a slow timescale with respect to the VS system (see Section III B), and in ITER the PF coils are superconductive, the following model can be considered when the VS loop is closed on the plant:

$$
\dot{\delta I}_{PF}(t) = (L^*)^{-1} \delta U_{PF}(t),
$$

(13a)

$$
\delta y(t) = C\delta I_{PF}(t).
$$

(13b)

Where the vector $\delta y(t)$ contains the plasma current plus a set of geometrical descriptors which completely characterize the plasma shape. The matrix $L^*$ PF system inductance matrix modified by the presence of the VS loop.

In this case we consider as geometrical descriptor 30 gaps plus the two strike-points, while the number of actuator in ITER is equal to $n_{PF} = 11$. Hence, a XSC-like approach (see Section IV C) has been adopted to design both plasma current and shape controllers.

Let us consider the following partition of the output vector $\delta y(t) = \left( \delta g^T(t) \quad \delta I_p(t) \right)^T$, where $\delta g(t)$ is the plasma geometrical descriptors vector, and $\delta I_p(t)$ is the plasma current. If $\delta g(t) = C_g \delta I_{PF}(t)$, we consider the following SVD

$$
C_g = U_g \Sigma_g V_g^T,
$$

(14)
and the control law is chosen equal to

\[
\delta U_{PF}(t) = K_{SF} \delta I_{PF}(t) + K_{P1} V_g \Sigma_g^{-1} U_g^T \delta g(t) + K_{I1} V_g \Sigma_g^{-1} U_g^T \int_0^t (\delta g(t) - \delta g_r(t)) dt \\
+ k_{P2} \delta I_p(t) + k_{I2} \int_0^t (\delta I_p(t) - \delta I_{pr}(t)) dt ,
\]

(15)

where \( \delta g_r(t) \) and \( \delta I_{pr}(t) \) are the reference on the plasma geometrical descriptors and the plasma current, respectively. The term \( K_{SF} \delta I_{PF}(t) \) is added in (15) so as to avoid the divergence of currents in the superconductive PF coils.

As for JET XSC, even for the ITER tokamak it turned out that some singular values – depending on the configuration – are one order of magnitude smaller than the others. Hence, even in this case, a trade-off condition is achieved controlling to zero only the error for the linear combination related to the largest singular values.

The proposed design approach has been proven to be effective in designing the plasma current, position and shape controller for ITER [27]. For example, Fig. 13 to Fig. 15 show the performance of the controller in tracking the shape reference shown as solid line in Fig. 12.

**Conclusions**

In this paper the three fundamental axisymmetric magnetic control problems for tokamak experimental devices have been introduced, namely the plasma vertical stabilization problem, the plasma shape control problem, and the plasma current control problem. The plasma magnetic control systems currently operated at the JET tokamak, together with a possible solution for plasma axisymmetric magnetic control in ITER, have been presented as examples.

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[16] G. D. TOMMASI et al., in *Proc. of the joint 48th IEEE Conf. on Decision and Control and 28th Chinese Control Conf.* (Shanghai, P.R. China, 2008), pp. 303-308.
[33] A proof of the need to add a poloidal magnetic field to confine the plasma in a tokamak can be found in [32, section 1.1.4]
[34] If $R_{\text{max}}$, $R_{\text{min}}$, $Z_{\text{max}}$, and $Z_{\text{min}}$ are, respectively, the maximum and minimum values of the $R$ and $Z$ coordinates along the plasma boundary (see Fig. 1), then the elongation is defined as

$$\kappa = \frac{Z_{\text{max}} - Z_{\text{min}}}{R_{\text{max}} - R_{\text{min}}}.$$ 

It follows that for plasmas with circular cross section it is $\kappa = 1$, while for elongated plasmas it is $\kappa > 1$.

[35] A model is said to be plasmaless if it is obtained in absence of plasma.
Figure 1: ITER tokamak cross-section and Poloidal Field (PF) coils system. The Central Solenoid (CS) coils are part of the PF system.
Figure 2: Plasma boundary descriptors. This figure shows the strike points and the X-point, together with gaps typically controlled on the JET tokamak. Note that a gap is not strictly the distance between the plasma surface and a point on the wall, but rather the distance measured on a fixed line. This definition simplifies the calculation and provides a good linear relationship between the currents in the poloidal field coils and the geometrical descriptors.
Figure 3: Simplified electromechanical model with three conductive rings. Two rings are kept fixed and in symmetric position with respect to the $r$ axis and model the active surrounding coils. The third ring can freely move vertically and it models the plasma.

Figure 4: Stable equilibrium for the simplified electromechanical model depicted in Fig.3.
Figure 5: Unstable equilibrium for the simplified electromechanical model depicted in Fig. 3.
Figure 6: The JET poloidal field coils system. The P1 circuit includes the elements of the central solenoid P1EU, P1C, P1EL, as well as P3MU and P3ML. The series circuit of P4U and P4L is named P4, while the circuit that creates an imbalance current between the two coils is referred to as IMB. SHA is made of the series circuit of P2SU, P3SU, P2SL, and P3SL. The fast radial field circuit, termed FRFA, connects the P2RU, P3RU, P2RL, and P3RL, and its used by the VS system. The central part of the central solenoid contains an additional circuit named PFX. Finally the four divertor coils (D1 to D4) are driven separately each by one power supply.

Figure 7: Possible architectures for plasma shape control.
Figure 8: Block diagram of the JET shape controller. The feedback selector allows to change among the available control modes.

Figure 9: JET Pulse No: 68953 the XSC has been used to control the plasma shape. Figure 9(a) shows a comparison between the measured shape and the reference at $t = 6.5s$, while the traces in Figure 9(b) show the time evolution of six plasma shape descriptors.
Figure 10: Block diagram of the eXtreme Shape Controller.

Figure 11: The JET VS system block diagram. The VS controller is composed of three parts. The velocity loop removes the plasma unstable pole. The current loop keeps the current in the actuator close to zero and operates on a slower timescale. The adaptive controller slowly changes the controller gains.

Figure 12: Tracking of a given shape. The reference shape is shown as solid line, while the dashdot and the red dash lines show the initial and final (after 25s) shape, respectively. Note that the plasma-wall distance on the outboard varies from ~6cm to ~15cm.
Figure 13: Tracking of a given shape. Mean square error on the controller plasma shape descriptors.

Figure 14: Tracking of a given shape. Time traces of the PF currents variations. These required currents for shape control are within the allowable limits of ~8kA.

Figure 15: Tracking of a given shape. In-vessel coil current time trace. Note that the maximum required current (~70kA) is within the limit of ~100kA envisaged for the ITER design review.