Modelling Neutral Particle Analyzer Measurements of High Energy Fusion Alpha-Particle Distributions in JET

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ABSTRACT

Measurements of high energy alpha-particle distribution functions using a neutral particle analyzer (NPA), in conjunction with measurements of the alpha-particle source profile, will be used to study the physics of fusion product generation and confinement in the forthcoming deuterium-tritium experiments in the Joint European Torus (JET). This paper addresses two key questions: the interpretation of the NPA measurements in terms of the time-evolving alpha-particle velocity distribution within the plasma, and the identification of the physical processes, additional to isotropic birth, confinement and classical slowing-down, which must be incorporated into theoretical models linking the measured alpha-particle source to the measured alpha-particle population. The NPA measurement can be identified with a vertical line integral through the plasma centre of the local alpha-particle velocity distribution. The latter quantity is computed rapidly by integrating along the characteristics of a simplified Fokker-Planck equation in which the alpha-particle source and plasma collisionality are empirical inputs. The source term is obtained from fusion reactivity profiles which are deduced in turn from tomographic measurements of 14 MeV neutron emission. Preliminary results, obtained using an idealized source term, indicate that the shape of the line-integrated alpha-particle energy spectrum can differ significantly from the local spectrum at the position of maximum fusion reactivity (the plasma centre), particularly when sufficient time has elapsed for a slowing-down distribution to become established. On the other hand, at energies close to the birth energy and at times (after the onset of alpha-particle production) shorter than the slowing-down time, the line-integrated and local energy spectra closely resemble each other. Application of the model is demonstrated for NPA measurements of fusion alpha-particle distribution functions in the Tokamak Fusion Test Reactor (TFTR).

1. INTRODUCTION

A high energy neutral particle analyzer (NPA) deployed on the Joint European Torus (JET) has been used successfully to measure the energy distribution of ions heated by waves in the ion cyclotron range of frequencies (ICRF) [1-3]. Specifically, the NPA measures a vertical line integral through the plasma centre of the heated ion energy distribution weighted by the local neutralization probability per unit time. In a previous paper [3], a simple relation was established between the best-fit temperature $\tilde{T}_\perp$ of the line-integrated distribution perpendicular to the magnetic field and the local perpendicular tail temperature in the plasma centre $T_\perp(0)$. When this analysis was applied to NPA measurements of ICRF-heated plasmas in JET, it was found that the scaling of $T_\perp(0)$ with ICRF power $P_{RF}$ was almost exactly linear, as predicted in a model of ICRF heating developed by Stix [4].

In this paper, prompted by the forthcoming programme of deuterium-tritium experiments on JET (DTE1), we assess the information that could be obtained on charged fusion products, in
particular alpha-particles, if the NPA were used in conjunction with neutron emissivity profile measurements [5]. Clearly, the neutralization of alpha-particles and the consequent formation of helium atoms which could be detected by the NPA, requires either double charge-exchange or sequential single charge-exchange reactions: in either case, the neutralization probability is very small. Nevertheless, the NPA has already been used to measure MeV energy ICRF-heated helium-3 ions in JET experiments, in which the required double charge-exchange reactions were made possible by injecting helium-4 atomic beams into the plasma[6]. Subsequently, the same NPA instrumentation has been used to measure fusion alpha-particles in the Tokamak Fusion Test Reactor (TFTR) [7]. In this case an ablating lithium pellet was used to produce a sufficiently high density of hydrogen-like and helium-like ions for a detectable flux of helium-4 atoms to be formed. The pellet charge exchange method is not currently available on JET, but helium-4 atomic beam injection may be used during DTE1 experiments, and it is possible that double charge-exchange with helium-like ions of intrinsic plasma impurities could also produce a detectable flux of neutralized fusion alpha-particles: single charge-exchange with hydrogen-like impurity ions is the dominant process responsible for neutralization of MeV ICRF-heated protons in JET [1,2].

For present purposes, let us assume that the total alpha-particle neutralization probability in the energy range of the NPA (0.3-4.0 MeV) is known, and flux to the NPA is sufficiently large for the fusion alpha-particle distribution to be determined in that energy range. In order to fully exploit the NPA for alpha-particle measurement, it is necessary first to consider the relation between the NPA-measured distribution function and the local alpha-particle distribution, as in Ref. [3]. We address this issue in Section 2. A model must then be constructed to link the local alpha-particle population to the alpha-particle source profile and plasma collisionality, which can be determined empirically. For reasons of analytical tractability, we adopt a simplified Fokker-Planck model to calculate the alpha-particle distribution: this model is used in Section 3 to simulate the NPA-measured alpha-particle distribution function. In Section 4, using data from TFTR, we give a simple demonstration of how the model could be used in practice to infer alpha-particle distribution function parameters from NPA measurements. In Section 5 we conclude that NPA measurements, combined with measurements of the time-evolving neutron emissivity profile [5], should yield useful information on the physical process governing alpha-particle dynamics in the JET DTE1 experiments.

2. LINE-INTEGRATED ALPHA-PARTICLE DISTRIBUTION

The NPA measures the energy distribution of atoms whose velocity vectors lie inside a narrow cone, centred on the vertical direction [1-2]: if the velocity components parallel and perpendicular to the vertical direction are $v_Z$ and $v_\perp$, only atoms with $v_Z/v_\perp \geq 200$ can be detected. Thus, the first step towards simulating an NPA alpha-particle spectrum is to establish a relation between
$f_\alpha$, the local alpha-particle velocity distribution, and $F_\alpha$, the local energy distribution of alpha-particles which, if neutralized, would be detected by the NPA (the alpha-particle energy is not significantly affected by the capture of two electrons). The relation between $f_\alpha$ and $F_\alpha$ is determined by the geometry of the NPA velocity-space cone referred to above. In the case of ICRF-heated minority ions, Eq. (4) and Eq. (5) of Ref. [3] indicate that the distribution function of ICRF-heated minority ions within the NPA velocity-space cone is essentially independent of $v_\perp$, and thus depends only on $v_Z$. This result is equally true of fusion alpha-particle distributions. It can be shown that

$$F_\alpha(E, Z) = \frac{\pi v_Z}{200^{2} m_\alpha} f_\alpha(v_Z, Z),$$

(1)

where $E = m_\alpha v_Z^2 / 2$, $Z$ is vertical distance above the midplane, and $m_\alpha$ is the alpha-particle mass. The factor $1/200^2$ reflects the small solid angle subtended in velocity space by the detector, while the weighting with $v_Z$ reflects the measurement of a particle flux, as distinct from an *in situ* distribution [3]. If $f_\alpha$ is symmetric about $Z=0$ we can define a line-integrated distribution by writing

$$\tilde{F}_\alpha(E) = \frac{2\pi v_Z}{200^2 m_\alpha} \int_0^b f_\alpha(v_Z, Z) dZ,$$

(2)

where $b$ is the height of the plasma. As stated in Section 1, the NPA measures a line integral of the alpha-particle distribution, weighted by the local neutralization probability per unit time $P_V(E, Z)$. Thus, the flux measured by the NPA is proportional to a quantity

$$\int_0^b P_V(E, Z) F_\alpha(E, Z) dZ$$

In Refs. [1-2], a proton energy distribution $F_{\alpha NPA}^\alpha(E)$ was obtained by dividing the NPA flux measurements by $P_V(E, 0)$, the neutralization probability per unit time in the plasma centre. Thus, we have

$$F_{\alpha NPA}^\alpha(E) = \frac{2}{P_V(E, 0)} \int_0^b P_V(E, Z) F_\alpha(E, Z) dZ$$

(3)

Now, the integral on the right hand side of Eq.(3) divided by $\tilde{F}_\alpha(E)$ [defined in Eq.(2)] can be regarded as an average neutralization probability along the NPA line-of-sight, weighted with respect to the local alpha-particle distribution. Denoting this average by $\overline{P}_V(E)$, it is clear that

$$\tilde{F}_{\alpha NPA}^\alpha(E) = \frac{\overline{P}_V(E)}{P_V(E, 0)} \tilde{F}_\alpha(E).$$

(4)
Since the average neutralization probability along the line-of-sight is not necessarily close to the neutralization probability in the plasma centre, we cannot be certain that $\tilde{F}_\alpha(E)$ is an accurate measure of the NPA-inferred spectrum. If, however, the local alpha-particle energy distribution is more strongly-peaked in $Z$ than is $P_x(E, Z)$, it follows then that $F_{\alpha NPA} \equiv \tilde{F}_\alpha(E)$.

3. LOCAL ALPHA-PARTICLE DISTRIBUTION

We now consider the problem of evaluating $f_\alpha$. Several numerical Fokker-Planck models have been developed to describe the temporal and spatial evolution of alpha-particle distributions in tokamaks [8-10]. Here, however, we adopt the simplified analytical approach to the Fokker-Planck equation used by Sigmar in Ref. [11] to model fusion product populations:

$$\frac{\partial f_\alpha}{\partial t} = S_\alpha(v, r, t) + \frac{1}{\tau_s} \frac{1}{u^2} \frac{\partial}{\partial u} \left[ (u^3 + v_c^3) f_\alpha \right],$$  \hspace{1cm} (5)$$

where $S_\alpha$ is the rate of alpha-particle production per unit volume of phase space, $\tau_s$ is the slowing-down time, $v = \left( v_Z^2 + v_1^2 \right)^{1/2}$, $v_C$ is the so-called critical velocity, and $r$ is minor radial distance. Expressions for $\tau_s$ and $v_C$ can be found, for example, in Ref. [12]. Equation (5) is applicable when the alpha-particle speed lies between the background ion thermal speed and the electron thermal speed. In the energy range of the NPA, this condition is invariably satisfied close to the centre of JET plasmas. The differential operator on the right hand side of Eq. (5) includes only the effects of collisional slowing-down, not collisional diffusion. This is justified when the spread of alpha-particle energies exceeds the electron temperature [13]; again, this condition is invariably satisfied in a thermonuclear plasma [14]. For simplicity, we also neglect particle losses, quasi-linear diffusion resulting from velocity space instabilities, finite orbit width effects, and velocity space anisotropy. Such an approach has been used successfully to interpret the observed time evolution of ion cyclotron emission (ICE) driven by the fusion product population in TFTR [15]. We shall adopt this approach, minimizing the number of free parameters and also the level of mathematical sophistication, at every opportunity in this paper. It will be demonstrated $a\ posteriori$ that such a methodology is justified. It is reasonable to assume that $S_\alpha$ is isotropic in a strictly thermonuclear plasma; if, as in DTE1, high power neutral beams are used to provide auxiliary heating, $S_\alpha$ will be anisotropic. Physically, anisotropy of $f_\alpha(v, r, t)$ is determined to leading order by the initial imprint of an anisotropic source term, and by the constraints imposed by toroidal magnetic field geometry, because slowing-down on electrons dominates until an alpha-particle is slowed to epithermal energies, when pitch angle scattering by thermal ions becomes effective [4]. The degree of anisotropy of the source depends on centre of mass effects arising from the beam ion energy, the beam injection angle with respect to the magnetic field, and the fusion product species [16,17]. Equation (5) could be modified to allow
for anisotropy both through the form of $S_\alpha$ and by incorporating a term describing pitch angle evolution due to magnetic moment invariance. The equation remains analytically tractable when this is included. In the present paper, however, we adopt the approximation that $S_\alpha$ is isotropic.

Equation (5) describes the temporal evolution of $f_\alpha$ on a particular magnetic flux surface: $S_\alpha$, $\tau_S$ and $\nu_C$, which are all functions of $r$, do not vary over that flux surface. The assumed isotropy of $f_\alpha$ implies that it does not vary with poloidal angle. We can therefore replace $r$ with $Z$ in Eq. (5). The plasma is assumed to contain equal numbers of deuterium and tritium ions: $\nu_C$, defined as the alpha-particle speed at which collisions with electrons and ions occur with equal frequency, must be modified to take this into account. Trapped alpha-particles with energies close to the mean birth energy of 3.5 MeV undergo large radial excursions during the course of a banana orbit, and so finite orbit width effects should, in general, be taken into account. However, the JET NPA only detects trapped alpha-particles with pitch angles lying close to 90°. Such particles lie close to the point of bounce reflection: at this point, by definition, the local orbit width is zero, so that the corresponding flux surface is uniquely defined. Prior to detection, the particles will, of course, have traversed a region with a spatially-varying slowing-down time, and therefore Eq. (5) should, strictly speaking, include a term describing the movement of alpha-particles across flux surfaces. However, the alpha-particles spend much more time in the vicinity of the bounce reflection point than they do close to the midplane (where the orbit width is greatest) and so, for the present purpose of modelling analytically the collisional evolution of the alpha-particle distribution, it is justifiable to use the approximation of zero orbit width.

Equation (5) is an inhomogeneous first order partial differential equation whose formal solution can be readily obtained using the method of characteristics:

$$f_\alpha(v, Z, t) = e^{\frac{3}{\tau_S}} \int_0^t e^{-\frac{3}{\tau_S} \eta} S_\alpha(v', Z, \eta) d\eta. \quad (6)$$

where

$$v' = \left[ (v^3 + \nu_C^3) e^{\frac{3}{\tau_S} (t-\eta)} - \nu_C^3 \right]^{1/3}. \quad (7)$$

The slowing-down time depends on electron temperature $T_e$ and density $n_e$, while $\nu_C$ depends on $T_e$ only. To obtain Eq. (6), we have assumed that $\tau_S(Z)$, and hence $T_e$ and $n_e$, are time-independent. Thus, alpha-particle production is assumed to occur during a flat-top in electron temperature, and alpha-particle heating of electrons is assumed to be negligible on the timescale of interest. In addition, during the anticipated high fusion reactivity pulses in DTE1 in JET, both $T_e$ and $n_e$ will evolve such that $\tau_S$ will be approximately time-independent. An initial condition $f_\alpha = 0$ at $t = 0$ has also been imposed. To evaluate $f_\alpha$ using Eq. (6), it is necessary to specify $S_\alpha(v, Z, t)$, $\tau_S$ and $\nu_C$ as functions of $Z$. We adopt the following frequently-used model profiles for $T_e$ and $n_e$:

$$T_e(Z) = T_{e0} \left(1 - Z^2/b^2 \right)^{\nu_{Te}} \quad \text{and} \quad n_e(Z) = n_{e0} \left(1 - Z^2/b^2 \right)^{\nu_{Ne}} \quad (8)$$
Typically, $v_{Te} > v_{ne}$. It is convenient to choose a representation of the source $S_{\alpha}$ which is separable in $Z$, $v$ and $t$, i.e. we assume that there exist functions $\zeta$, $\psi$ and $H$ such that $S_{\alpha} = \zeta(Z)\psi(v)H(t)$. The functions $\zeta$ and $H$ can be obtained experimentally from tomographic analysis of 14 MeV neutron flux measurements [5]: such measurements will be used as input in applications of our approach. Neutron emission time profiles in both the JET preliminary tritium experiment [5] and in deuterium-tritium pulses in TFTR [18] generally exhibit a gradual rise and decay, with the flat-top (if any) being short compared to the rise and fall times. For the purposes of analytical tractability we assume a step function time profile: thus, we set $H = 1$ for $0 < t < \tau$, and $H = 0$ otherwise. To model the space and velocity dependence of $S_{\alpha}$, we assume that the reacting deuterons and tritons have Maxwellian distributions with equal temperatures $T_i(Z) = T_{i0}\left(1 - Z^2/b^2\right)^{\nu_{ni}}$. In that case, we can write [11, 14]

$$\zeta(Z) = \frac{1}{4}n_i^2(\sigma v) = \zeta_0 \frac{n_i(Z)^2}{T_i(Z)^{2/3}} \exp\left[-\frac{20}{T_i(Z)^{1/3}}\right]$$

where $n_i(Z) = n_{i0}\left(1 - Z^2/b^2\right)^{\nu_{ni}}$ is the total ion density, $\sigma$ is the reaction cross-section, angled brackets denote expectation values, $T_i$ is in keV, and $\zeta_0$ is a normalization parameter whose value is determined by the requirement that the integral of $\zeta$ over the volume of the tokamak be equal to the total alpha-particle production rate per unit time, which we denote by $S_{\alpha}$. The integral of $\psi$ over velocity space must then be unity. Brysk [14] obtained an expression for the energy distribution function of neutrons in a thermonuclear plasma: the same expression, with the neutron mass and birth energy replaced with the corresponding alpha-particle parameters, can be assumed to be a reasonable description of the velocity dependence of $S_{\alpha}$. Adapting Brysk’s result, we obtain

$$\psi(v) = \frac{\exp\left[-(v^2 - v_{\alpha}^2)^2/\delta v^4\right]}{2\pi^{3/2}v_{\alpha}\delta v^2},$$

where $v_{\alpha} \equiv 1.3\times10^7$ ms$^{-1}$ is mean alpha-particle birth speed and $\delta v = \left[8v_{\alpha}^2T_i/(m_n + m_{\alpha})\right]^{1/4}$, where $m_n$ and $m_{\alpha}$ are the neutron and alpha-particle masses. In evaluating the normalization factor in $\psi$, we have made the reasonable assumption that $\delta v^2 \ll 2v_{\alpha}^2$. For the purpose of evaluating $\delta v$, we neglect the spatial variation of $T_i$ by setting it equal to $T_{i0}$.

There are realistic limiting cases in which the integral in Eq. (6) can be evaluated analytically, for example, letting $\delta v \to 0$, to describe alpha-particles created with a unique speed $v_{\alpha}$, we recover the expression for $f_{\alpha}$ obtained by Sigmar [11]:

$$\exp\left[-(v^2 - v_{\alpha}^2)^2/\delta v^4\right]$$
where

$$ t' = \frac{\tau_S}{3} \ln \left( \frac{v^3 + v_c^3}{v^3 + v_c^3} \right) $$

Equation (11) has been used by the present authors to model the time evolution of fusion products generating ICE in TFTR [15]. In another limiting case describing a short duration pulse of fusion reactivity, we let $\tau \to 0$ and obtain

$$ f_\alpha(v, Z, t) = \frac{\tau_S}{2\pi^{3/2}v_\alpha \delta v^2} \exp \left[ - \frac{\left( (v^3 + v_c^3) e^{3\tau_S} - v_c^3 \right)^{2/3} - v_\alpha^2}{\delta v^4} \right] $$

for $t > \tau$. In the general case of finite $\delta v$ and $\tau$, the integration in Eq. (6) must be performed numerically. Equations (11) and (13) provide useful benchmarks for this general case.

For illustration we have computed $f_\alpha$ numerically for $\tau = 5s$ and $\tau = 0.1s$, representing fusion reactivity pulses of, respectively, long and short duration compared to $\tau_S(Z = 0)$. The following parameter values, appropriate for deuterium-tritium pulses in JET, were assumed: $n_{e0} = n_{i0} = 2x10^{19}$ m$^{-3}$, $v_{ne} = v_{ni} = 1$, $T_{e0} = 10$ keV, $T_{i0} = 20$ keV, $v_{Te} = v_{Ti} = 2$, major radius $R_0 = 3.1m$, and $S_\alpha = 4x10^{18}$ s$^{-1}$. With these parameters, the slowing-down time in the plasma centre $\tau_S(0)$ is approximately 1.9s. The plasma height $b$ is typically 1.6m in JET, but $S_\alpha$ falls off sufficiently rapidly with vertical distance $Z$ that we need only evaluate $f_\alpha$ for $Z \leq 1.2m$. Three-dimensional plots of $f_\alpha$ at different times are shown in Fig.1 ($\tau = 5s$) and Fig.2 ($\tau = 0.1s$). For $\tau = 5s$, the distribution at a given $Z$ is filled in from the high energy end on a timescale determined by $\tau_S$. This varies as $T_e^{3/2}/n_e$, which is a monotonic decreasing function of $Z$. The low energy part of the distribution is therefore filled in more rapidly at large $Z$ than it is at $Z=0$. This has the effect of inverting temporarily the spatial profile of $f_\alpha$ at constant $v$ [see, in particular, Fig. 1(c)]. It should be stressed that this does not arise due to spatial diffusion, which is omitted from our model: it arises rather from the spatial dependence of $\tau_S$. In the case of $\tau = 0.1s$, the distribution at a given $Z$ does not broaden in velocity with time: after the end of fusion reactivity, all the alpha-particles slow down at essentially the same rate. It is apparent from Fig. 2 that the area under the velocity distribution at $Z = 0$ continues to increase with time at $t > \tau$. However, the alpha-particle density on each flux surface, which is proportional to the integral over $\nu$ of $\nu^2 f_\alpha$, remains constant during the slowing-down process. In both Fig. 1 and Fig. 2, the alpha-particle pressure, which is proportional to the integral over $\nu$ of $\nu^4 f_\alpha$, remains strongly peaked in the plasma centre at all times, despite the transient spatial inversion of $f_\alpha$ noted above.
Fig. 1. Alpha-particle distribution $f_{\alpha}(Z, v_Z)$ for long duration ($\tau = 5s$) fusion reactivity pulse at (a) $t = 0.5s$, (b) $t = 1.0s$, (c) $t = 2.0s$, and (d) $t = 3.0s$. The plasma parameters are the following: $n_{e0} = n_{i0} = 2 \times 10^{19} m^{-3}$, $v_{ne} = v_{ni} = 1$, $T_{e0} = 10$ keV, $T_{i0} = 20$ keV, $v_{Te} = v_{Ti} = 2$, $b = 1.6m, R_0 = 3.1m$, and $S_{\alpha} = 4 \times 10^{18} s^{-1}$. The slowing-down time in the plasma centre is $\tau_S(0) \approx 1.9s$.

Fig. 2. Alpha-particle distribution $f_{\alpha}(Z, v_Z)$ for brief duration ($\tau = 0.1s$) fusion reactivity pulse at (a) $t = 0.1s$, (b) $t = 0.2s$, (c) $t = 0.4s$, and (d) $t = 1.0s$. The plasma parameters are the following: $n_{e0} = n_{i0} = 2 \times 10^{19} m^{-3}$, $v_{ne} = v_{ni} = 1$, $T_{e0} = 10$ keV, $T_{i0} = 20$ keV, $v_{Te} = v_{Ti} = 2$, $b = 1.6m, R_0 = 3.1m$, and $S_{\alpha} = 4 \times 10^{18} s^{-1}$. The slowing-down time in the plasma centre is $\tau_S(0) \approx 1.9s$.

The solid curves in Fig. 3 and Fig. 4 represent the line-integrated energy spectra $\tilde{F}_\alpha(E)$ corresponding to the distributions shown in Fig. 1 and Fig. 2. The broken curves represent the local energy distributions in the plasma centre $F_{\alpha}(E, 0)$, scaled in each case to have the same maximum value as $\tilde{F}_\alpha(E)$. In general, the solid and broken curves closely resemble each other, indicating that the shape of the line-integrated distribution is a good approximation to that of the local distribution. This is particularly true early in the pulse, and if $S_{\alpha}$ is of short duration [see, for example, Fig. 4(a)]. Before a slowing-down distribution becomes fully established [as in Figs. 3(c) and 3(d)], the line-integrated distribution peaks at a slightly lower energy than $F_{\alpha}(E, 0)$, and the discrepancy between the two distributions is greater on the low energy side of the peak than it is on the high energy side. This is due mainly to the strong spatial dependence of the slowing-down time noted above. If we assume that $\tilde{F}_\alpha(E)$ is a good approximation to the measured NPA spectrum, it follows that the latter can be regarded as an accurate measure of $F_{\alpha}(E, 0)$ during the early stages of collisional evolution. In the case of Fig. 4 ($t = 0.1s$), the measured alpha-particle flux is predicted to be negligible at $t \geq 1s$: by this time, essentially all the alpha-particles have slowed down to energies below 0.3 MeV.
We consider finally the case of a pulse with a higher central density, and hence a shorter slowing-down time. Figs. 5 and 6 show respectively $f_\alpha$ and $\tilde{F}_\alpha(E)$ for a parameter set which is identical to that used to generate Figs. 1-4 except that $t = 1.5s$, $n_{e0} = n_{i0} = 5x10^{19} \text{ m}^{-3}$, and hence $\tau_S(0) \approx 0.74s$. In the energy range of the NPA, a slowing-down distribution becomes established 1s after the onset of alpha-particle production. After the end of alpha-particle production, there is a rapid fall in the simulated NPA signal at high energy, but not at low energy. Consequently, the spectrum becomes progressively steeper at $t > \tau$.

4. APPLICATION TO TFTR DATA

As a simple illustration of how the model proposed here could be used to infer distribution function parameters from NPA alpha-particle spectra, we apply it to measurements on TFTR. Fig. 6 of Ref. [19] shows two alpha-particle energy spectra, measured close to the centre of...
TFTR immediately after periods of tritium neutral beam injection (NBI), lasting for $\tau$=1s and $\tau$=0.1s. In the latter “beam blip” case (pulse #86299), the spectrum is fairly narrow and peaks at an energy of about 2.5 MeV, with no signal below 1.7 MeV. The measurements were obtained at $t=0.12s$ after the start of NBI (i.e. 0.02s after the end of NBI), at which time $n_0=6\times10^{19} m^{-3}$, $T_e(0)=6$ keV, and hence $\tau_S(0) \cong 0.58s$. The pellet injection method of providing electron donors used in TFTR ensures that the NPA measurement gives the local alpha-particle energy distribution, rather than the line-integrated distribution [6]. Thus, the TFTR spectrum can be identified with the quantity $F_\alpha(E, Z)$ defined by Eq. (1), with $Z \equiv 0$. We have used the analytical Sigmar model, described above, to evaluate $F_\alpha(E, Z)$ for TFTR pulse #86299. In this case the pulse duration $\tau$, although shorter than the slowing-down time, is too large for Eq. (13) to be applicable, so that we have evaluated the integral in Eq. (6) numerically. Most of the required
parameters are listed above: we assume that alpha-particle production occurred only during the period of neutral beam injection, and that \( S_\alpha = 2 \times 10^{14} \, \text{s}^{-1} \). For the purpose of comparing model predictions of the alpha-particle spectrum shape with TFTR measurements, the exact value of \( S_\alpha \) is not important. In any case, due to uncertainty in the neutralization probability of alpha-particles during pellet injection in TFTR, the absolute magnitude of the measured distribution is itself uncertain [19]. The only remaining free parameter is \( T_i_0 \). The equivalent temperature of thermalized beam ions in deuterium-tritium supershot pulses is typically around 100 keV at energies below the beam energy \( E_0 \), and around 10-15 keV at energies above \( E_0 \) [20]. In general, the energy-space width of the measured alpha-particle spectrum is certain to be affected by the fact that the beam ions have a much higher mean energy than the bulk deuterium ions. In fact, the plasma betas of beam ions and thermal ions in TFTR supershot pulses are typically comparable [20]. The mean effective temperature of the deuterium and tritium ions in TFTR pulse #86299, obtained from neutron spectroscopy measurements, was estimated in Ref. [19] to be 30 keV: accordingly, for the purpose of evaluating \( f_\alpha(u,0) \), we set \( T_i_0 \) equal to 30 keV.

The quantity \( F_\alpha \) in Fig. 7 is the number of alpha-particles per unit volume per unit energy propagating in all directions: it is equal to the distribution defined by Eq. (1), except for the geometrical factor \( \pi/200^3 \) discussed in Section 2. The data points, obtained using the NPA, have been normalized to have the same maximum value as the predicted distribution. Our simplified model is in good agreement with the measured distribution, in terms of both peak energy and energy-space width. The peak energy is a rather sensitive function of the slowing-down time, and hence \( T_e_0 \) and \( n_e_0 \), despite the fact that \( t \) and \( \tau \) are smaller than \( \tau_S \). In fact, for the parameter regime of pulse #86299, it can be shown from Eq. (6) that the peak energy differs from the mean birth energy \( E_\alpha \) by an amount \( \Delta E \sim 2(t/\tau_S)E_\alpha \approx n_e/T_e^{3/2} \).

In Fig. 8 the alpha-particle energy distribution given by Eq. (1) is plotted for three values of \( T_e_0 \); the other parameters are identical to those used to generate the curve in Fig. 7. It is clear that the predicted spectrum is sensitive to \( T_e_0 \) and that good agreement with the measured energy distribution is obtained for \( T_e_0 = 30 \) keV.

Fig. 7. Points: measured alpha-particle energy distribution in the centre of TFTR, immediately after a short pulse (duration \( \tau = 0.1s \)) of tritium beam injection (pulse #86299). Curve: distribution given by our treatment of the Sigmar model, with \( Z = 0, \tau = 0.1s, t = 0.12s, n_e_0 = n_i_0 = 3 \times 10^{19} \, \text{m}^{-3}, v_{ne} = v_{ni} = 1, T_e_0 = 6 \) keV, \( T_i_0 = 30 \) keV, \( v_{Te} = v_{Ti} = 2, b = 0.8 \text{m}, R_0 = 2.52 \text{m}, \text{and} \ S_\alpha = 2 \times 10^{14} \, \text{s}^{-1} \). The data points have been normalized to have the same maximum value as the predicted distribution. The quantity \( F_\alpha \) is the number of alpha-particles per unit volume per unit energy, propagating in all directions: it has been evaluated from the local velocity distribution \( f_\alpha \) using Eq. (1), with the geometrical factor \( \pi/200^3 \) omitted.
spectrum in pulse #86299 is only achieved if we set \( T_{e0} \) equal to its actual value, namely 6keV. In Ref.[19], the alpha-particle distribution of pulse #86299 was simulated using a fully numerical Fokker-Planck code FPPT, in combination with TRANSP: these simulations reproduced successfully the NPA-measured alpha-particle distribution. We have demonstrated here that the essential features of the TFTR alpha-particle distribution measurements can be obtained using a very simple analytical slowing-down model.

The assumption of an isotropic birth profile of the Brysk type [Eq. (10)] may be oversimplistic in the case of high power NBI-heated plasmas. However, the analytical approach of the model proposed here makes it possible, in principle at least, to infer directly alpha-particle distribution function parameters from NPA spectra. Furthermore, as we noted in the previous section, the model remains analytically tractable when anisotropy is included. In that sense, our method of calculating \( f_\alpha \) complements the fully numerical approach employed in Refs. [8-10] and Ref. [19]. It should be emphasized that the solution given by Eq. (6) can be rapidly evaluated for any source function \( S_{\alpha}(v, Z, t) \), and, furthermore, can be generalized in a straightforward way to take into account alpha-particle losses [11]. Toroidal magnetic field ripple losses, for example, make the alpha-particle distribution more centrally peaked than would otherwise be the case [7]. This effect increases the likelihood of \( F_\alpha \) being more centrally peaked than the neutralization probability: when this condition is satisfied, the line-integrated spectrum \( \tilde{F}_\alpha \) can be regarded as an accurate representation of the NPA spectrum.

5. CONCLUSIONS

Using parameters corresponding to deuterium-tritium experiments in JET, we have simulated alpha-particle energy spectra which could be measured using the JET NPA. We have assumed that the NPA measurement can be identified with a vertical line integral through the plasma centre of the local alpha-particle distribution \( f_\alpha(v, Z, t) \). The latter quantity can be computed rapidly, by integrating along the characteristics of a simplified Fokker-Planck equation. Three-dimensional plots indicate that the low energy part of the distribution is filled in more rapidly far from the plasma centre than it is at \( Z = 0 \), and therefore the spatial profile of \( f_\alpha \) can be strongly
dependent on alpha-particle energy. In particular, there is a transient spatial inversion of $f_\alpha$ at low energy, although the alpha-particle pressure profile remains strongly peaked in the plasma centre at all times. The spatial inversion can be attributed to the fact that the slowing-down time decreases monotonically with distance from the plasma centre. Consequently, the shape of the line-integrated alpha-particle energy spectrum can differ significantly from the local energy spectrum at the position of maximum fusion reactivity (the plasma centre). On the other hand, at energies close to the birth energy and at times (after the onset of alpha-particle production) shorter than the slowing-down time, the line-integrated and local energy spectra closely resemble each other. We have shown that the model can account for the essential features of an alpha-particle spectrum measured on TFTR. The simplicity (and hence analytical tractability) of the model means that it should be possible to infer alpha-particle distribution function parameters directly from NPA measurements in JET.

Given a set of measurements of the time-evolving fusion reactivity profile, and the parameters needed to determine local collisionality, we can use the model to predict the NPA measurement of alpha-particle distribution, if the key physical processes incorporated in the Sigmar model (with its many simplifications) are dominant, as appears also to be the case for fusion product-driven ICE in TFTR [15]. These processes include isotropic birth, confinement, and classical slowing-down. Isotropic losses, although not included in the specific calculations presented here, can also be incorporated into the model [11]. Conversely, substantial deviation of the NPA measurement from that predicted by the model would be evidence that other physical effects not included in it, for example collective effects or velocity space anisotropy, were playing a significant role in alpha-particle dynamics.

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