

Evaluation of Epsilon-Net Calculated Equilibrium Reconstruction Error Bars in the EUROfusion WP-CD Platform

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Evaluation of epsilon-net calculated equilibrium reconstruction error bars in the European Integrated Modeling (EU-IM) Platform

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Abstract

An important direction of fusion research is the reconstruction of plasma equilibrium from measurements. Most of the plasma modeling codes and plasma control systems use equilibrium data on input. Therefore, the accuracy of reconstructions plays a crucial role in fundamental understanding of processes in present devices and fusion reactors. The results of previous research show that one can get substantially different reconstructions of plasma current densities and safety factors, which fit the measurements even within a relatively small inaccuracy. So, rigorous calculation of the reconstructed functions error bars is required. This paper presents new advances in formulation of the equilibrium reconstruction problem, describes application of the ε -nets technique for rigorous calculation of the reconstructions error bars and its software implementation, gives examples of error bars evaluation for ASDEX Upgrade.

Keywords: equilibrium reconstruction, ill-posed problems, epsilon-nets, error bars

I. Introduction

One of the most essential fields in fusion research is the reconstruction of plasma equilibrium. Reconstructed data is used as input to most of the plasma modeling codes and to plasma control systems. Therefore, the accuracy as well as the associated uncertainties of reconstructions play a crucial role in fundamental understanding of processes in present devices and fusion reactors such as ITER and DEMO. The results of previous and recent research, see Refs. 1-3 and references therein, show that the traditional methods of equilibrium reconstructions can only be trusted with a rigorous calculation of the reconstructed functions error bars. One can get substantially different plasma current density and safety factor profiles associated to plasma equilibria that fit the diagnostic measurements obtained even within a relatively small inaccuracy. However, it is hard to find equilibrium reconstructions published with rigorous calculating of their error bars.

A recently developed new technique, based on ε -nets and SDSS code¹⁻³, allows addressing the fundamental challenges of the equilibrium reconstruction problem:

- (a) calculate the reconstructed functions error bars;
- (b) with preset inaccuracy ε find all solutions of the inverse equilibrium problem, which fit the measurement errors, including very different solutions;
- (c) find the efficiency of a diagnostic (constraint) in selecting a reconstruction appropriate to the real physical process;
- (d) determine the required accuracy of the measurements for a diagnostic;
- (e) validate the effect of advancements in plasma model, used for reconstruction, on the error bars magnitude, such as: anisotropic pressure, loop voltage, Ohm's law for plasma evolution, iron core, passive elements, 3D equilibrium.
- (f) design real time feedback automatic plasma control systems, based on reconstruction of plasma boundary and internal parameters with the ε -net technique.

Here we concentrate on item (a).

In this paper new advancements in formulation of the general equilibrium reconstruction problem are proposed, software implementation of the numerical method is described and new results on rigorous reconstruction error bar analysis with ε -nets for ASDEX Upgrade equilibrium are presented.

II. Advancements in formulation of the inverse problems

The solution of the general problem of the equilibrium reconstruction can be separated into two stages: determining the plasma boundary (external problem) and then reconstructing the current density inside the found plasma boundary (internal problem). Such separation is of advantage, since the external problem is less complicated and can be solved separately with higher accuracy.

The detailed formulation of the external and internal problems together with numerical algorithms for their solution is given in Refs. 1, 2 and with more details in sections 1.8.1, 2.4, 2.5, 4.10 of Ref. 3. Here we describe only new advancements in formulations, using notation of Refs. 1-3.

In the external problem we introduce inequalities (constraints)

$$\begin{aligned} |\psi(t, R_k, Z_k) - \psi_k(t)| / |\psi_k(t)| &\leq \delta_{\psi,k}, \\ \left| \frac{\partial \psi}{\partial \mathbf{n}_k}(t, R_k, Z_k) - \frac{\partial \psi_k}{\partial \mathbf{n}_k}(t) \right| / \left| \frac{\partial \psi_k}{\partial \mathbf{n}_k}(t) \right| &\leq \delta_{\partial \psi / \partial \mathbf{n}, k}, \end{aligned} \quad (1)$$

instead of equation (3) of Ref. 2. Here $\psi(t, R, Z)$ is the poloidal flux to be reconstructed, (R_k, Z_k) are cylindrical coordinates of the k -th sensor, ψ_k is given measured poloidal flux at the sensor, $\partial\psi_k/\partial\mathbf{n}_k$ is given measured directional derivative, $\delta_{\psi,k}$ and $\delta_{\partial\psi/\partial\mathbf{n},k}$ are known inaccuracies of measurements. New constraint (1) involves more details of the magnetic diagnostics in formulation of the external problem than the one used in Refs. 2, 3.

In Ref. 2 the internal problem was formulated using inequality (11) for normal to the plasma boundary derivative of ψ . This inequality contains the inaccuracy δ of the poloidal field specification at the plasma boundary. The value of δ is calculated in the external problem (see example at the end of section VII). In order to avoid involvement of inaccuracy δ we replace constraint (11) of Ref. 2 with constraint (1). This directly inputs information from measurements to the internal problem, but somewhat complicates its formulation and solution.

Before, all data from the external problem entered the internal one only through quantities $\Phi = RB_{pol}$ and δ in constraint (11), Ref. 2, calculated in the external problem. This constraint was set locally at the plasma boundary. Now, we have the internal problem with non-local at the plasma boundary constraint (1), in which ψ is determined by process inside and outside the plasma. Function ψ in (1) should be accurately expressed through quantities, searched in the internal problem, known from the external one and measured experimentally.

The solution of the Grad-Shafranov equation for given toroidal current density $j(t, R, Z)$ can be expressed at any point (R_k, Z_k) with the help of the Green function of the Grad-Shafranov operator (see, for example, section 1.3.5.2 of Ref. 3)

$$\psi(t, R_k, Z_k) = \iint_{S_{chm}} \mu_0 j(t, R, Z) G(R_k, Z_k, R, Z) ds, \quad (2)$$

where S_{chm} is the area of the vacuum chamber in the vertical section (in (R, Z) plane). Region S_{chm} consists of the plasma area $S(t)$, vacuum area between the plasma and the chamber walls $S_v(t)$, and the area of the chamber walls S_w (where magnetic sensors are located).

Splitting integral (2) in the sum of integrals over these three areas, we present $\psi(t, R, Z)$ at a sensor (R_k, Z_k) in (1) with the sum of the poloidal fluxes

$$\psi(t, R_k, Z_k) = \psi_S(t, R_k, Z_k) + \psi_c(t, R_k, Z_k) + \psi_w(t, R_k, Z_k), \quad (3)$$

produced respectively by toroidal currents in plasma, in poloidal field coils (PFC) and solenoid, and in passive elements and vessel walls.

The poloidal flux from the plasma $\psi_S(t, R_k, Z_k)$ can be expressed through the triplet (ψ_S, p, F) inside the plasma, which is searched in the internal problem, (see, for example, sections 1.3.5.2, 2.3.2 and 2.5.3 of Ref. 3)

$$\begin{aligned} \psi_S(t, R_k, Z_k) &= \iint_{S(t)} \mu_0 j_\eta(t, R, Z) G(R_k, Z_k, R, Z) ds \\ &\approx \mu_0 \sum_i j_\eta(t, R_i, Z_i) G(R_k, Z_k, R_i, Z_i) \Delta S_i(t), \end{aligned} \quad (4)$$

where $\Delta S_i(t)$ is the area around point (R_i, Z_i) of the toroidal annulus filament and $j_\eta(t, R, Z)$ is the toroidal plasma current density

$$j_\eta(t, R, Z) = R \frac{\partial p(t, \psi_S)}{\partial \psi_S} + \frac{1}{2\mu_0 R} \frac{\partial F^2(t, \psi_S)}{\partial \psi_S}. \quad (5)$$

The poloidal flux from the control coils ψ_c is defined by currents in PFC and solenoid $J_c(t, R_{c,i}, Z_{c,i})$

$$\begin{aligned} \psi_c(t, R_k, Z_k) &= \iint_{S_v(t)} \mu_0 J_c(t, R, Z) \delta(R - R_{c,i}, Z - Z_{c,i}) G(R_k, Z_k, R, Z) ds \\ &= \mu_0 \sum_i J_c(t, R_{c,i}, Z_{c,i}) G(R_k, Z_k, R_{c,i}, Z_{c,i}). \end{aligned} \quad (6)$$

The poloidal flux ψ_w is similarly given by currents J_w induced in walls and passive elements

$$\psi_w(t, R_k, Z_k) = \mu_0 \sum_i J_w(t, R_{w,i}, Z_{w,i}) G(R_k, Z_k, R_{w,i}, Z_{w,i}). \quad (7)$$

If currents J_w are small in comparison with other currents or if the plasma is far away from the walls, then contribution of ψ_w in (3) is small and can be neglected. Otherwise, for better accuracy of the reconstruction it is preferable to have currents J_w measured experimentally and used as input to both the external and the internal problem. If currents J_w are important, but not measured, then they should be calculated.

Modeling of currents J_w in the considered two stage formulation can be also done in two stages. In the external problem one can avoid J_w in summation over all currents in equation (2), Ref. 2 and account walls only in constraint of the constant flux (5), Ref. 2. In the internal problem J_w can be calculated by one of the approaches discussed in section 1.3.6 of Ref. 3 and used in formula (7) to find ψ_w . During solution these two stages can be iterated, introducing J_w from the previous iteration in sum (2), Ref. 2, to get better consistency between J_w and constraint (5) of Ref. 2.

Alternatively one can consider the external and internal problems simultaneously and solve them in one stage with ε -nets technique at the expense of increase in computational time. For example, with each plasma boundary $(R(\xi), Z(\xi))$, one can solve the internal problem equations (7)-(10), (12)-(15) of Ref. 2, then find J_w (section 1.3.6 of Ref. 3) and solve the external problem with constraint (1) using J_w in sum (2), Ref. 2.

In this paper, besides the constraint (1), we also study the constraint for plasma pressure. Introduction of this constraint supposes that plasma pressure $p_{msr}(t, \psi_S)$ is known from measurements with inaccuracy δ_p

$$\| (p(t, \psi_S) - p_{msr} p(t, \psi_S)) / p_{msr} p(t, \psi_S) \| \leq \delta_p. \quad (8)$$

Reconstruction error bars accounting (8) were not investigated before.

In some studies, the experimental values of measured ψ_k and $\partial\psi_k/\partial\mathbf{n}_k$ can be replaced by numerically generated ones. Values of ψ_k and $\partial\psi_k/\partial\mathbf{n}_k$ can be set from equations (3)-(7) with given (p, F) and given plasma boundary. Plasma boundary and (p, F) are known, for example, in the problem of the error bars analysis for the given (target) equilibrium, obtained from the solution of the direct problem or some reconstruction problem. If error bars are to be

found for the target plasma boundary only and (p, F) are not given, then ψ_k and $\partial\psi_k/\partial\mathbf{n}_k$ can be generated from ψ satisfying the external problem constraints with substitution of the target plasma boundary in them.

In case of artificially generated ψ_k and $\partial\psi_k/\partial\mathbf{n}_k$ the formulation and solution of the inverse problem can be simplified by omitting fluxes ψ_w and ψ_c in setting ψ_k and not using them in equations (1) and (3). In such approach ψ_w and ψ_c are considered as given and thus cancel in constraints (1), since they enter the total poloidal flux (3) additively. Therefore, in such a problem there is no necessity to consider currents J_c and J_w .

III. Error bars calculation

By the error bar interval (confidence interval) of some target function we understand the strip of minimal width around it, which contains all reconstructions that fit inaccuracies of diagnostic measurements.

It is important to note that, in essence, error bars exist mainly due to many different equilibriums, which all fit even very accurately measured data and not necessarily due to a particular regularizing method used for solving the inverse reconstruction problem. Thus, the width of the error bars is determined mainly by the formulation of the problem, i.e. constraints used.

The ε -nets algorithm allows finding all equilibriums which fit measurements, see Refs. 1, 2 and sections 1.8.1, 2.4, 2.5.7, 4.10 of Ref. 3. Therefore, in the ε -nets approach, the boundary of the confidence interval at each point is simply formed by minimum and maximum values over all ε -net elements, left in the ε -net after application of all of the constraints.

The inaccuracy of the error bars calculations in the ε -nets method is given by the inaccuracy ε , see Refs. 1-3. In probabilistic approaches there is uncertainty about the inaccuracy. So, at present the ε -nets technique is the only one, which allows *rigorous* calculation of the reconstruction error bars.

Some understanding of the reconstruction problem output data stability on the input data can be obtained by randomizing the input parameters, for example, spline coefficients in p and F . But there is no guarantee that all solutions of the reconstruction problem, which fit the measurements, are found with such randomization. Thus, the band around the reconstruction, obtained with the randomization approach, can be associated with the error bars interval only with some probability or as some lower estimate of the error bars.

IV. Code SDSS

The numerical algorithms are implemented in the code SDSS (Substantially Different Solutions Searcher) in Fortran 2008, Refs. 1-3. The code has the graphic user interface (GUI), written in Java, to help setting up the input data, constructing ε -nets, solving the inverse problem, visualizing and analyzing the results. The software size of SDSS and GUI is ~25000 lines.

The algorithm is fully parallelizable⁴. SDSS has options for MPI/CPU and OpenCL/GPU calculations. MPI gives close to maximum possible scalability over the number of CPUs acceleration. The OpenCL/GPU option can additionally speed up in ~10-100 times on one GPU. Real time equilibrium reconstructions are possible⁴, which are especially important for designing feedback automatic plasma control systems.

Depending on the ε -net size used, the required computing power is from a PC for ~10⁸ elements to a super-computer for ~10¹⁰-10¹³. Each process requires ~100 Mb RAM. Total HD space necessary is ~500 Mb.

SDSS allows finding p' and FF' error bars with inaccuracy $\varepsilon \sim 10\%$ in ~ 3 minutes on 32 CPUs for ε -net of splines over 15 points in the normalized poloidal flux $\rho \in [0,1]$ with $3 \cdot 10^4$ elements for $dp/d\rho$ and $3 \cdot 10^4 dF^2/d\rho$, giving $9 \cdot 10^8$ different right sides for the Grad-Shafranov equation to be solved. Finding plasma boundary error bars in the external problem is much faster.

Additional optimization of performance can be done by calculating different typical ε -net sets once for a particular tokamak and then using them routinely for solving inverse problems.

V. European Integrated Modeling Platform

Code development and integrated modeling are key players in fusion roadmap, since they provide the main tools for understanding plasma behavior in present and future devices.

General views on the main issues and special features of software development in fusion are presented in detail in chapter 3 and afterword of Ref. 3. The development of a comprehensive fusion integrated modeling analysis suite is usually a rather long process, requiring many efforts from specialists in different branches of science, including physicists, mathematicians, programmers and engineers.

In the framework of activities of the WP-CD (Code Development for Integrated Modeling Work Package) project in the European Consortium for the Development of Fusion Energy (EUROfusion), a comprehensive suite of codes, validated on present devices and ready for ITER and DEMO predictions is being integrated on the EU integrated modeling platform⁵ (EU-IM).

The elemental features of the integrated modeling infrastructure are:

- Common data ontology, composed of CPOs⁶ (Consistent Physical Objects), which provide a structural representation of all physical quantities and technical objects in tokamaks and also provide interfaces to data storage methods. In addition, codes within the EU-IM platform communicate with each other only through CPOs.
- Kepler graphical user interface workflow engine to manage/orchestrate scientific workflows (www.kepler-project.org). In Kepler, the user describes computational dataflow simply by inserting boxes (the Actors, corresponding to physics modules), connecting them with each other, and setting control structures like “if then else”, “switch”, “loop” and etc. The execution of Actors can be sequential or parallel. There is possibility to have nested Actors, which in turn can contain their own workflows. Actors are presented by a dedicated software that essentially consists of a Java wrapper on top of the physics code library, enabling I/O to CPOs from within Kepler (Java based) and Kepler Actor life-cycle methods.
- Multiple native source code: Fortran 90 or higher, C++, Python, Java, Matlab. For all of these languages special interfaces and routines exist that allow a code to communicate (get/put) the CPOs with databases.

The ε -nets theory and its implementation in code SDSS are rather sophisticated. So, it is important to put efforts in making SDSS available for a seamlessly routine use by the fusion community. This is especially significant for support of data processing, validation and analysis, and for design of diagnostics and control systems for ITER and DEMO reactors. At present, SDSS is being implemented in the EU-IM platform with support from EUROfusion (in the frame of WP-CD project) to allow for a routine analysis with ε -nets of the reconstruction error bars for practically any tokamak device.

VI. Reconstruction error bars in ASDEX Upgrade

ASDEX Upgrade plasma pulse #25374 has been considered at $t = 2.5$ s. This pulse, at this particular time, is characterized by a plasma elongation ≈ 1.77 , minor radius ≈ 0.5 m, magnetic axis at ≈ 1.7 m, magnetic field at the magnetic axis ~ 2 T, total toroidal current ≈ 0.8 MA and a normalized plasma beta ≈ 1.1 .

The problem consisted of finding the error bars for equilibrium reconstruction performed by code CLISTE⁷⁻⁹. The ε -nets approach, described in sections II-V, was applied. Calculations were done with SDSS code in the EU-IM environment. Magnetic measurements inaccuracies in equations (1) and other data required for SDSS input were obtained directly from the appropriate CPOs. The inaccuracies have the order of $\delta_{\psi,k} \sim 1\%$, $\delta_{\partial\psi/\partial\mathbf{n}_k} \sim 1\%$. The poloidal flux ψ_k and directional derivative $\partial\psi_k/\partial\mathbf{n}_k$ at sensors were generated synthetically, as discussed in the end of section II, using CLISTE equilibrium reconstruction. Plasma pressure was assumed to be available with inaccuracy $\pm 5\%$: $\delta_p = 5\%$. Constraints of Motional Stark Effect (MSE), polarimetry, interferometry, the scrape-off layer currents and separatrix behavior near the X-point can be easily incorporated in the problem (see Ref. 2), but were not used here in order to allow pure study of the pressure constraint (8) and the role of the directional derivatives constraint in (1). Joining all the constraints can reduce error bars.

Reconstruction error bars for plasma boundary and internal plasma parameters are shown in Figs. 1-3. Dashed curves give CLISTE's (target) reconstruction examined with ε -nets technique. Bars present the error bar strip, in which all reconstructions (solutions of the problem described in section II) that fit measurements are contained. The inaccuracies of error bars calculation in the considered case are $\varepsilon < 1\%$ for the external problem and $\varepsilon \approx 7\%$ for the internal one.

One can see that error bars of plasma boundary reconstruction are $\sim 3\%$, which is close to the magnetic measurements inaccuracy $\pm 1\%$. Error bars for toroidal current density and safety factor are ~ 10 - 20% (worse for q at the plasma center). These have the order of measurements inaccuracies: $\pm 5\%$ for p and $\pm 1\%$ for ψ and $\partial\psi_k/\partial\mathbf{n}_k$. Error bars for p' are less than $\pm 5\%$ as expected due to p' ε -net bounds taken, see solid curves in Fig. 3. Error bars for FF' are noticeably worse, but FF' component of the toroidal current density is much less and doesn't impact much j_η and q . The origin of difficulties in separating p' and FF' is studied thoroughly in item 2, section 2.5.3, Ref. 3.

Special runs for both external and internal problems with only the first constraint in (1) or with only the second one showed that measurements of $\partial\psi_k/\partial\mathbf{n}_k$ are much more informative, since reduce plasma boundary and j_η error bars in ~ 30 - 40% .

In order to find the relation between alternative formulations of the problem, with constraint (1) or with constraint (11) of Ref. 2, we calculated the left side of inequality (11), Ref. 2 on ε -nets elements that fit constrain (1) and found that $\delta_{\psi,k} \approx 1\%$, $\delta_{\partial\psi/\partial\mathbf{n}_k} \approx 1\%$ is approximately appropriate to $\delta \approx 6\%$. This additionally confirms correctness of values of δ used in Refs. 1-3.

Also, error bars studies for ASDEX Upgrade with magnetic measurements alone were performed. This gave unacceptably high error bars for j_η , q , p' , FF' as it was in all cases considered for ITER, JET and MAST in Refs. 1-3.

VII. Conclusions and further developments

The radically new method for equilibrium reconstruction was considered. The method is based on the ε -nets technique. It allows solving a variety of problems which were very hard to address before: mathematically rigorous calculating the reconstruction error bars; validating the existence or absence of very different solutions, which are compatible with the same measurement errors; studying the efficiency of a constraint in selecting a solution; evaluating

the required accuracy of the measurements; validation of the effect of advancements in the plasma model used; design real time feedback automatic control systems.

The magnetic diagnostics together with some data about the vacuum chamber can provide plasma boundary reconstruction with reasonable inaccuracy. However, it was confirmed again that magnetic diagnostics alone are not sufficient in the internal problem for finding one reconstruction appropriate to the real process.

It is demonstrated that well measured plasma kinetic pressure can be an efficient additional constraint for selecting the correct reconstruction. So, from the results of this paper and Refs. 1-3 one can make a well-grounded conclusion that at least three efficient constraints exist for determining real internal plasma state: Motional Stark Effect (MSE), kinetic pressure and polarimetry. However, p' and FF' error bars can be not satisfactory even with these. Advancement of the plasma model used for reconstruction, see item (e) of the introduction, may help to improve accuracy for p' and FF' .

It was found that $\partial\psi_k/\partial\mathbf{n}_k$ diagnostics is noticeably more efficient than ψ_k measurements.

It is clear from this paper, Refs. 1-3 and other publications that equilibrium reconstructions should always be published together with error bars, like other diagnostic data.

The ε -nets toolset and the SDSS code are being incorporated in the EU-IM software environment to allow for a seamlessly routine analysis of the reconstruction error bars for any tokamak device.

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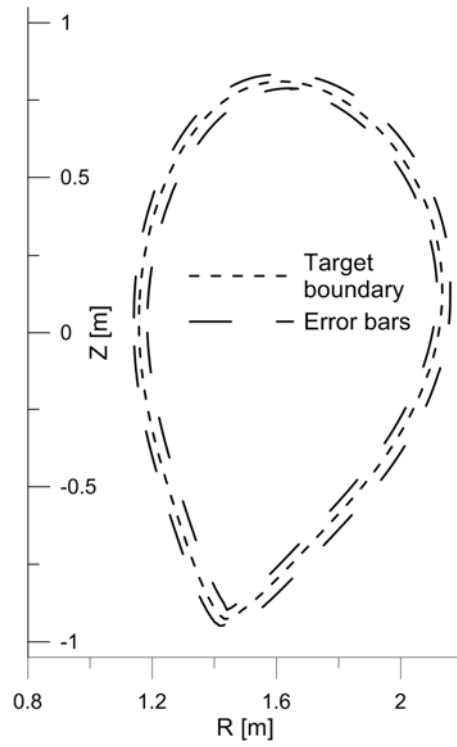


Figure 1. Vertical section of ASDEX Upgrade plasma in pulse #25374. Short dashed curve shows plasma boundary reconstructed with code CLISTE. Plasma boundary error bars are calculated with code SDSS using ϵ -nets technique.

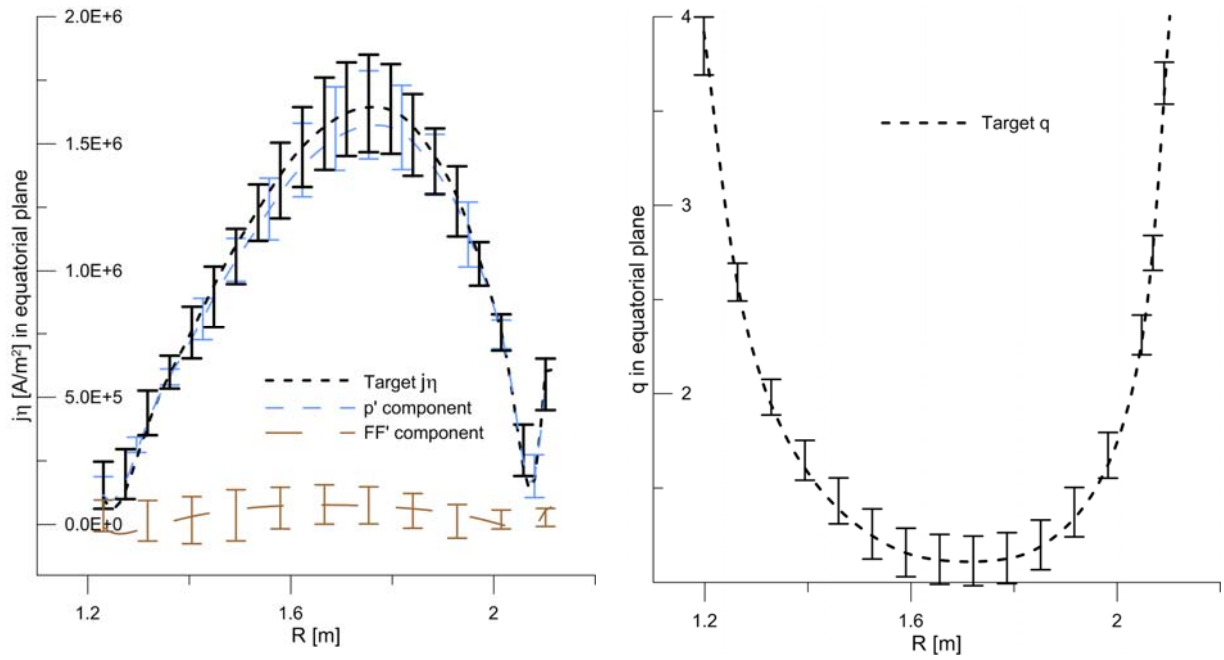


Figure 2. The ASDEX Upgrade plasma, pulse #25374. Plasma current density and its components (left), and safety factor (right) at $Z=0$, reconstructed with code CLISTE, and error bars, calculated with code SDSS using ε -nets technique.

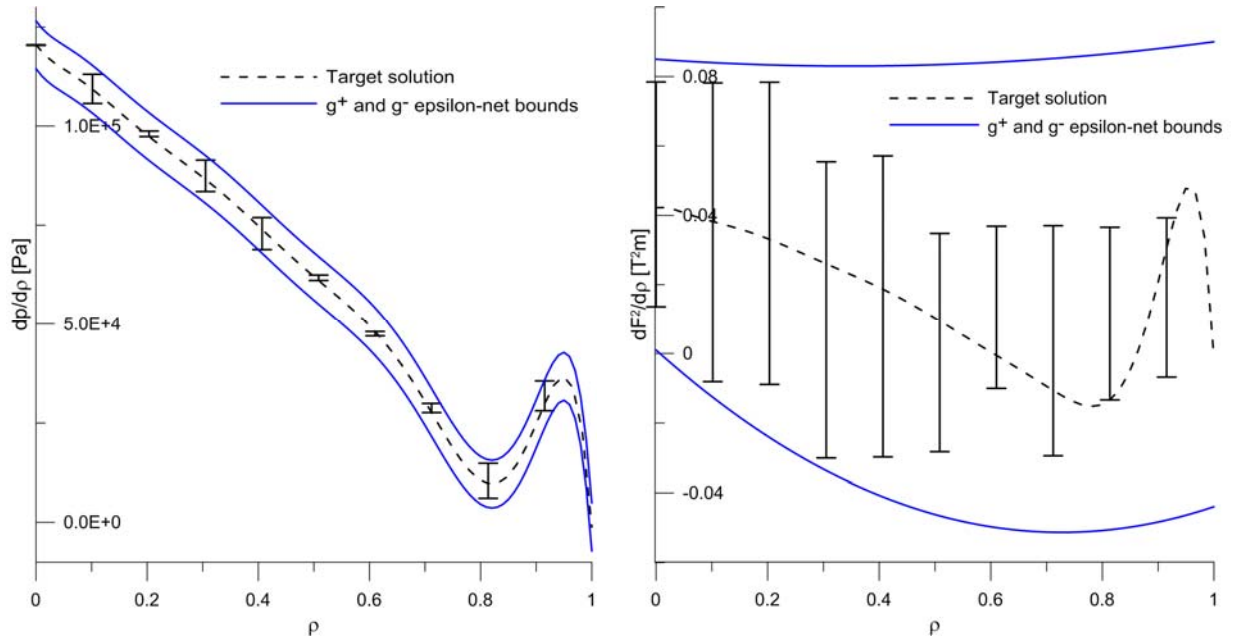


Figure 3. The ASDEX Upgrade plasma, pulse #25374. Reconstructed components of the toroidal current density \mathbf{j}_η (code CLISTE) with the error bars (code SDSS) as functions of the normalized poloidal flux $\rho \in [0, 1]$. Upper and lower solid curves show bounds of the region, in which ε -nets were constructed.