Role of MHD dynamo in the formation of 3D equilibria in fusion plasmas
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Abstract. This work shows via experiments and modelling that dynamo, or magnetic flux pumping mechanisms play a key role in the formation of helical core equilibria in fusion plasmas. Dynamo effects impact the current density profile and thus the safety factor profile of the final 3D equilibrium, with important consequences on MHD stability and transport. We compare results from multiple machines (RFX-mod, MST, DIII-D) and nonlinear MHD modelling. Two paradigmatic cases of helical state formation are considered and common physics is identified: spontaneous formation in high-current reversed-field pinch (RFP) plasmas and the hybrid scenario in high-β tokamak plasmas. Helical cores form in both cases, either spontaneously via saturation of MHD modes, or due to the marginally-stable ideal MHD response to external 3D fields. Direct measurements of the dynamo e.m.f. associated to 3D plasma distortions are presented for a database of helical RFP plasmas from RFX-mod and MST, covering a wide range of plasma parameters. Similar measurements were also made in helical states forming in response to external 3D fields in Ohmic RFX-mod tokamak plasmas and in DIII-D high-β hybrid plasmas. These experimental results qualitatively agree with nonlinear MHD modelling performed with the SpeCy1 and PIXIE3D codes. They indicate that central current is redistributed by a dominantly electrostatic MHD dynamo. The underlying physics common to RFP and tokamak is thus revealed: a helical core displacement modulates parallel current density along flux tubes, which requires a helical electrostatic potential to build up, giving rise to a helical dynamo flow.

1. Introduction

This work investigates the role of dynamo mechanisms in the formation and sustainment of self-organized, long-lived helical states in fusion plasmas, including both reversed-field pinches (RFP) and tokamaks, and discusses their common physics. We show that in both configurations self-organized helical states determine the mean current density profile through dynamo, or magnetic flux pumping mechanisms, with important consequences for the equilibrium, transport, and MHD stability properties of these plasmas. Nevertheless a validated dynamo model is not yet available. We offer here a possible interpretation of such effects as due to a dominantly electrostatic MHD dynamo, as suggested by a comparison of nonlinear magnetohydrodynamic (MHD) simulations and experiments in multiple machines and magnetic configurations.

Various observations of long-lived helical states exist in different types of laboratory and astrophysical plasmas, with interesting commonalities not yet fully investigated. In all cases
helical states result from the nonlinear saturation of an MHD instability. They can be often considered stationary with respect to the resistive diffusion time scale and they can be thus modelled as 3D equilibria. This work considers two examples of self-organized helical states in laboratory fusion plasmas, namely (a) helical states forming spontaneously during high-current RFP operation in the 1-2MA range and (b) helical states induced by external 3D magnetic fields in high-\( \beta \) hybrid tokamak plasmas. We introduce in the following the characteristics of these two types of plasmas that are relevant to this work.

In RFP plasmas, a spontaneous transition to a helical equilibrium was observed to occur with increasing probability and duration as the plasma current is increased \([1]\). This was reproduced in all RFP machines and could be thoroughly investigated in particular in RFX-mod, the only RFP device able to explore the high-current regime up to 2MA. The MHD mode that gives rise to these helical states is a core resistive kink-tearing mode with poloidal/toroidal mode numbers \( m=1/n=2R/a\approx7 \), due to the relatively low safety factor of these plasmas (here \( R \) and \( a \) are the major and minor radius of the torus). As this mode saturates into the helical state, the amplitude of multiple, \( m=1/higher-n \) modes resonant at outer radii keeps decreasing as the current is raised. Such modes cause significant magnetic chaos and large heat and particle transport in low-current RFP operation. But at high current above 1MA, due to their amplitude being significantly reduced and the partial formation of helical flux surfaces, an electron internal transport barrier forms around mid-radius. The necessity of dynamo effects to sustain RFP equilibria and in particular to drive poloidal currents and enable the toroidal magnetic field reversal near the edge was clear since first RFP experiments \([2, 3]\). Several experiments and significant modelling efforts demonstrated the important role of a turbulent MHD dynamo driven by multiple modes. Recent work focused on laminar MHD dynamo in helical states \([4]\), which has the strongest link with the tokamak experiments discussed here. More details on these mechanisms are given in section 2.

In tokamaks, saturated MHD modes are observed to cause current redistribution, which can have important consequences, in particular in the high-\( \beta \) hybrid scenario \([5]\). For example, in the DIII-D tokamak a saturated 3/2 tearing mode is responsible for broadening the central current profile through flux pumping mechanisms, keeping the central safety factor above unity \([6]\). This avoids \( n=1 \) activity like sawteeth and makes \( n=1 \) tearing modes more stable, allowing \( \beta \) values higher than in standard H-mode to be reached, with \( \beta_{S}=\beta_{T} aB_{T}/\mu_{r} \) up to 3.5-4. Flux pumping also makes possible fully non-inductive hybrid operation by efficiently redistributing significant co-current driven by electron cyclotron waves (ECCD) near the center, where ECCD is most efficient. Anomalous current redistribution is clearly present in these experiments, but a validated flux pumping model is not yet available, though it is needed for physics scaling towards future machines. Here we show that the MHD dynamo model developed for helical RFP states could also explain flux pumping in hybrid tokamak plasmas. The results of dedicated experiments and their MHD modelling will be presented.

The paper is organized as follows. Section 2 discusses the MHD dynamo model through results of nonlinear MHD simulations of helical states in RFP and tokamak. Section 3 shows how the MHD dynamo e.m.f. can be directly calculated for helical RFP equilibria reconstructed in experiment by 3D equilibrium codes like VMEC/V3FIT. MHD dynamo measurements in a large database of helical RFP experiments from the RFX-mod and MST machines are analyzed. Section 4 shows MHD dynamo measurements in low-current Ohmic tokamak plasmas in RFX-mod, which provide a quite simple situation to facilitate model validation in the tokamak case. Section 5 shows the results of experiments and modelling of MHD dynamo in high-\( \beta \) hybrid plasmas in DIII-D. Section 6 gives the main conclusions and future perspectives of this work.
2. Dynamo effects in nonlinear MHD simulations of RFP and tokamak helical states

The simplest dynamo model for fusion plasmas is provided by nonlinear resistive MHD. Including nonlinearities is essential for a self-consistent description of dynamo effects, since MHD modes modify the current profile, which in turn affects their saturation. The first theoretical investigation of a laminar MHD dynamo for stationary helical states was made for the RFP [4]. Here we summarize the main elements of this model and we extend the same approach to helical states in tokamaks.

The essential elements of the MHD dynamo model, common to RFP and tokamak, can be described in a rather simple way, based on nonlinear MHD simulations at zero β and cylindrical geometry. Additional effects due to finite β, toroidicity and shaping, or effects beyond single-fluid MHD, like Hall MHD dynamo or kinetic effects can be added with more efforts, though they are not expected to qualitatively modify the predictions of this model, as discussed in [7].

Figure 1 presents the results of a nonlinear MHD simulation of a single-helicity RFP plasma by the SpeCyl code, which we use here to introduce the MHD dynamo model. The helical distortion of the core flux surfaces is large and similar to experimental cases in MST and RFX-mod. It was recognized in [4] for the first time that the stationary helical distortion of the flux surfaces produces a modulation of the parallel current density along flux tubes, which is associated to a finite, though very small charge separation. As a result, a helical electrostatic potential φ builds up, whose gradient provides a dynamo e.m.f. that is represented in figure 1(a) in a poloidal cut. This e.m.f. has two main effects. First, a helical $E \times B$ flow is associated to the dynamo e.m.f., as shown in figure 1(c), which can be in principle measured in experiment and used to validate the model. Second, the component of the dynamo e.m.f. parallel to the total magnetic field (n=0 plus perturbed part) in the helical equilibrium has finite n=0 average. This n=0 contribution enters Ohm’s law and is responsible for the redistribution of parallel current. This is shown in figure 1(d), which compares the $E_0$ and $\eta J_0$ terms in Ohm’s law. Note in particular how the dynamo e.m.f. opposes the applied loop voltage in the plasma core, similarly to what happens in tokamaks, but it also drives finite parallel current in the edge region, where the parallel loop voltage goes to zero. In fact in RFPs, different from tokamaks, the edge parallel current is mostly poloidal and thus it cannot be driven by external induction.

Similar cylindrical simulations were done for β=0 tokamak plasmas to evaluate the dynamo e.m.f. and the associated flow produced by different modes, and in particular by the 1/1 internal kink and by a 2/1 tearing mode. A significant effort is also ongoing to include realistic geometry and finite-β effects for both RFP [8] and tokamak [9]. They may provide quantitative predictions and will be hopefully validated soon in experiment.
Figure 2. Nonlinear MHD simulation showing the effect of a saturated 1/1 internal kink on the current and safety factor profiles through the MHD dynamo e.m.f. in Ohm’s law.

Figure 3. Nonlinear MHD simulation showing the effect of a saturated 2/1 tearing mode on the current and safety factor profiles through the MHD dynamo e.m.f. in Ohm’s law.

Figure 2 shows the result of these calculations for the 1/1 internal kink in an Ohmic tokamak. The left column shows the helical electrostatic potential and the associated flow, which feature a typical double convective cell structure, together with the dynamo electric field parallel to the total magnetic field. The right column shows the radial profiles of the parallel current density, the safety factor, and Ohm’s law terms before (black) and after saturation of the 1/1 mode (red). Central current is redistributed outward and as a result the safety factor is elevated around or above unity. The parallel dynamo e.m.f. is responsible for this current redistribution, as also found in the RFP. In the same equilibrium, a 2/1 tearing mode was forced to grow instead of the 1/1 mode, by external 2/1 fields, since in this case the 2/1 mode is stable. A large 2/1 island develops and convective cells form, as shown in figure 3. The dynamo effect from the 2/1 mode is evident in the radial profiles in the right column: the 2/1 mode redistributes current around its rational surface, but it does not affect current away from it, showing that the effect is localized where the helical displacements occur. In toroidal, shaped plasmas multiple harmonics can couple and the total dynamo effect will include contributions from all of them, possibly being more global. The 2/1 mode can have a 1/1 core sideband due to finite $\beta$ and $q_{\text{min}}$ just above 1. Similarly the 3/2 mode can have a large 2/2 sideband. This may be directly responsible for the central current redistribution in hybrid plasmas, as suggested also in [10].

3. MHD dynamo calculations for experimental helical RFP equilibria

Experimental evidences of a helical flow with double convective cell structure compatible with expectations from nonlinear MHD simulations for helical RFP states were obtained both in MST [11] and RFX-mod [12] using Doppler spectroscopy.
Here we present a further important step towards the experimental validation of the MHD dynamo model for helical states. While nonlinear MHD calculations are necessary to model MHD dynamo self-consistently including its dynamics, it is important to note that the stationary MHD dynamo e.m.f. alone can be also obtained directly from a static 3D equilibrium. Experimental 3D equilibrium reconstructions of helical RFP states became recently available [13]. They are obtained using V3FIT, a code that iterates many runs of the VMEC 3D equilibrium code while minimizing the distance from a series of experimental constraints. A similar approach was also recently applied to DIII-D, as it will be shown in section 5. Once such a 3D equilibrium is available, it is possible to obtain the MHD dynamo e.m.f. by balancing Ohm’s law over its helical flux surfaces [7]. In general one can write under stationary conditions ($\nabla \times \vec{E} = 0$):

$$E_{\text{loop}} - \nabla \varphi = \eta_{\text{neo}}(j - j_{\text{CD}} - j_{\text{BS}}) - \nu \times B = \eta_{\text{neo}} j_{\text{Ohm}} - \nu \times B,$$

where $\varphi$ is the electrostatic potential, $j$ the total current from V3FIT, $j_{\text{CD}}$ the externally driven current (NBI, EC) and $j_{\text{BS}}$ the bootstrap current, which can be calculated with dedicated codes. The last two terms are not present in the helical RFP, but will be included for DIII-D high-$\beta$ plasmas. By projecting Ohm’s law along the total magnetic field, including helical perturbations, the following equation in straight-field-line coordinates is easily derived:

$$\frac{B \cdot \nabla \varphi}{B \cdot \nabla \theta} = \frac{B \cdot (E_{\text{loop}} - \eta_{\text{neo}} \Delta j)}{B \cdot \nabla \theta} = \partial_\theta \varphi + q \partial_\zeta \varphi,$$

where the only unknown is the electrostatic potential $\varphi$, which can be obtained by integrating the equation over helical flux surfaces, while all other terms can be obtained from the 3D equilibrium reconstruction or they can be easily calculated from experimental data. Figure 4(a) shows the result of this calculation for an experimental helical RFP equilibrium from RFX-mod. The electrostatic potential has a dipolar structure similar to that found in nonlinear MHD simulations and the associated flow structure is similar to that measured in experiment [12]. The effective loop voltage provided by the helical distortion is obtained by taking an $n=0$ average of the parallel dynamo e.m.f. and it is shown in figure 4(b). As expected for a dynamo e.m.f., the calculated effective $V_{\text{loop}}$ is negative in the core and amounts to a significant fraction of the applied $V_{\text{loop}}$, which is around 18V in these plasmas.

Figure 5 shows the helical flow amplitude measured as described in [11] vs the magnetic perturbation for a $m=1$ flow vs magnetic 1/7 perturbations at different plasma currents in RFX-mod and MST helical RFP states.
wide database of RFX-mod discharges at different currents, I_p=[0.2-1.8]MA. The flow and magnetic perturbation amplitudes are strongly correlated and increase with plasma current as expected. Similar measurements done in MST are also compared.

4. MHD dynamo helical flows measured in RFX-mod Ohmic tokamak plasmas

Similar measurements of the helical flow spatial structure were also recently made in helical equilibria obtained in RFX-mod low-current Ohmic tokamak plasmas. A helical core equilibrium with m=1/n=1 was obtained in low-q(a) plasmas by applying slowly-rotating n=1 fields with non-axisymmetric coils. The helical core is imaged by soft x-ray tomography and extends over a large portion of the minor radius due to the large q=1 radius at low-q(a), which facilitates Doppler spectroscopy flow measurements. These experiments offer a good starting point to validate the MHD dynamo model in tokamaks, given their simple shape and the absence of complications due to finite β and non-inductive currents.

Figure 6 shows the results of these measurements for a q(a)=1.8 RFX-mod discharge. Magnetic feedback controls the amplitude of the 2/1 resistive wall mode at a predefined level, in this case applying a linear ramp-up and ramp-down, as shown in figure 6(b). The finite 2/1 kink distortion induces by toroidal coupling a large 1/1 sideband, as confirmed by soft x-ray tomography (not shown). Similar results were also obtained by forcing directly the 1/1 mode, with q(a) both below and above 2.

The toroidal flow measured at mid-radius, approximately near the boundary of the helical core, shows a periodic oscillation in phase with the applied n=1 field, while its amplitude follows the externally applied n=1 field amplitude, as shown in figure 6(c). Similar results are observed for the m=1 poloidal flow component in figure 6(d), which is obtained by the difference of two similar lines of sight on opposite sides of the magnetic axis, as shown in figure 6(f). These results are qualitatively compatible with the predictions of nonlinear MHD simulations done with the SpeCyl code shown in figure 6(e,f) and already introduced above in section 2. The dynamo e.m.f. calculated from the flow and magnetic field perturbation measurements also agrees with expectations: it can be easily estimated as 0.2km/s×0.3mT=0.06V/m, which amounts to about 30% of E_loop=0.2V/m.

5. MHD dynamo predicted for high-β hybrid plasmas in DIII-D

Experiments were performed in DIII-D to validate the MHD dynamo model introduced above in high-β hybrid plasmas. Normally flux pumping in hybrids is provided by a 3/2 tearing mode that saturates at moderate amplitude causing only limited confinement degradation. Since this mode rotates with the plasma in the 20kHz range, internal measurements for this mode with existing diagnostics, for example internal magnetic field measurements based on the Motional Stark Effect (MSE), are not yet possible given their typical temporal resolution.
To avoid these limitations, the following approach was adopted. Previous ASDEX Upgrade experiments to probe the plasma response to external n=1 fields in high-β hybrid plasmas found that the n=1 response is dominated by the 1/1 core harmonic, due to the flat q profile lying just above 1 [14]. In other terms, a helical core equilibrium with displacements of a few cm forms. DIII-D reproduced this result and used it to probe flux pumping mechanisms in hybrids. This approach has the advantage that the externally induced helical core can be slowly rotated at a predefined frequency, which enables internal measurements from various diagnostics, including soft x-ray, MSE, and others.

To isolate the helical core contribution to flux pumping, the 3/2 mode in a standard hybrid plasma was suppressed by ECCD. Simultaneously an n=1 field was applied with internal non-axisymmetric coils (I-coils) to induce the helical core. Figure 7 shows two such discharges with (red) and without (black) n=1 fields applied. When the 3/2 mode is suppressed in absence of a helical core, sawteeth reappear, indicating loss of hybrid conditions, as documented before [15]. No n=1 sawtoothing activity is observed when the helical core is induced, suggesting that this may provide enough flux pumping to maintain hybrid conditions. This interpretation is actually complicated by the presence of an n=3 tearing mode, which could provide some level of flux pumping.

To better quantify these effects, a measure of flux pumping in these and other repeat discharges was done using a method developed in [16] to estimate the consumption inside the plasma of poloidal flux provided by external coils. Recent work has shown that the rate at which poloidal flux is dissipated in hybrid plasmas is larger than the rate at which it is externally supplied, with this deficit being proportional to the amount of flux pumping [17]. Justified by this result, we applied the method and found that cases without 3/2 and without helical core have small or no flux consumption deficit, while plasmas with the helical core have finite flux consumption deficit. This result was used to estimate the equivalent effective loop voltage due to the helical core, which amounts to about 10mV, independent of the n=3 mode, which is present in all these discharges.

The internal measurements of the helical core distortions collected during these experiments, and in particular multi-chord soft x-ray and MSE measurements covering the entire outer midplane and extending up to the core, as recently reported in [14], were used to obtain the first 3D equilibrium reconstruction in a tokamak by the V3FIT code, as done before in RFX-mod. More details are being published in a dedicated paper [18]. The helical core equilibrium fitted in these plasmas was used to calculate the MHD dynamo e.m.f. with the same approach described for helical RFP states in section 2. The only difference is that in high-β hybrid plasmas the contribution of NBI, ECCD, and bootstrap current must be properly calculated and subtracted from the total current provided by V3FIT, as explained above. The result of this calculation is shown in figure 8. Panel (a) shows the helical flux surfaces provided by the V3FIT equilibrium and the helical electrostatic potential calculated in this way. A zoom of the same quantities in the plasma core is shown in figure 8(b), where the associated E x B flow is also shown (black arrows), which features the same double convective cell structure discussed above. The effective V_loop in figure 8(c) is negative in the core and positive outside and can produce the required current redistribution. A value of a few 10mV is compatible with the
above flux pumping estimate obtained independently, indicating that the MHD model provides a reasonable picture of flux pumping in hybrid plasmas.

6. Conclusions and future perspectives

Different experiments were performed to obtain helical states in RFP and tokamak plasmas and quantify their impact on the current profile through dynamo mechanisms. A relatively simple dynamo model based on nonlinear MHD was developed and compared against experiment. A full validation of this model needs both more modelling and experimental efforts. Quantitative predictions in particular require finite-\( \beta \), realistic geometry, and tearing modes to be included, which is the aim of ongoing work. Internal 3D measurements of the helical flow structure would be also necessary to measure the full \( \mathbf{v} \times \mathbf{B} \) term in Ohm’s law, in particular for fast-rotating modes, though such measurements exceed present diagnostic capabilities. Even with the present limits, these results indicate that the MHD model is a good candidate to explain present observations in different conditions and magnetic configurations.

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